A comparative study of assumption-based approaches to reasoning with priorities.

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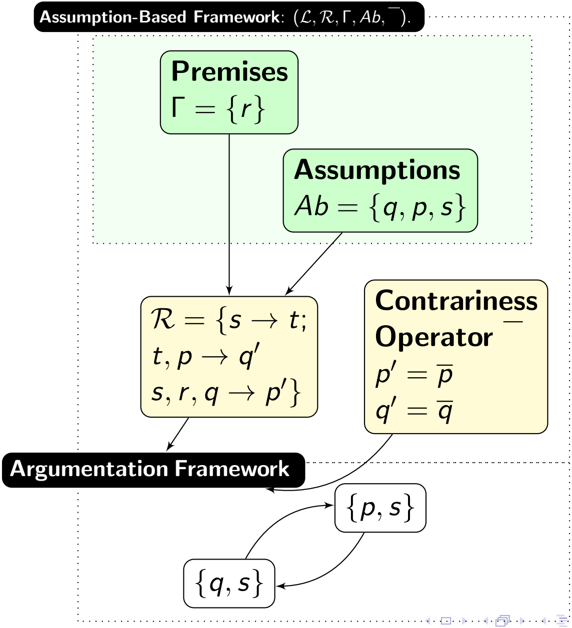
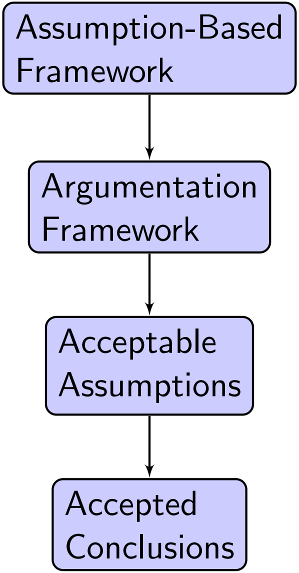
The Plan

Assumption-Based Argumentation

Assumption-Based Argumentation



The Argumentation Pipeline



Assumptions-based Frameworks

Definition (Assumption-based framework)

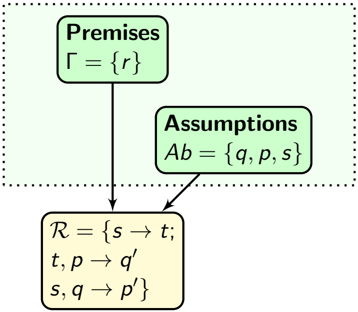
An assumption-based framework is a tuple $\mathtt{ABF} = (\mathcal{L}, \mathcal{R}, Ab, \overline{}, v)$ *where:*

- $\mathcal{L}$ *is a formal language*
- $\mathcal{R}$ *is a set of rules*
- $\emptyset \neq Ab \subseteq \mathcal{L}$ *is a (finite) set of candidate assumptions.*
- $\overline{} : Ab \to \mathcal{L}$ *is a contrariness operator.*
- $v : Ab \to \mathbb{N}$ *is a function assigning natural numbers to the assumptions.*

Flat Frameworks

We will additionally assume that frameworks are flat, i.e.
$A_1, \ldots, A_n \to A \notin \mathcal{R}$ for $A \in Ab$.

Deductive System $(\mathcal{L}, \mathcal{R})$



Example

- $\{s\} \vdash_{\mathcal{R}} t$
- $\{s, p\} \vdash_{\mathcal{R}} q'$

Definition ($\mathcal{R}$-deduction)

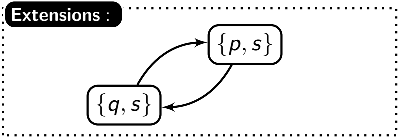
An $\mathcal{R}$-deduction from Δ of A, written $\Delta \vdash_{\mathcal{R}} A$, is a finite tree where

1. *the root is A,*
2. *the leaves are either empty nodes or elements from Δ,*
3. *the children of non-leaf nodes are the conclusions of rules in $\mathcal{R}$ whose antecedent correspond to their children,*
4. *Δ is the set of all $A \in Ab$ that occur as leaves in the tree.*

Attacks

Example

- $Ab = \{p, q, s\}$.
- $\mathcal{R} = \{q \to \overline{p}; p \to \overline{q}\}$
- $\{q\} \vdash_{\mathcal{R}} \overline{p}$.
- $\{p\} \vdash_{\mathcal{R}} \overline{q}$.



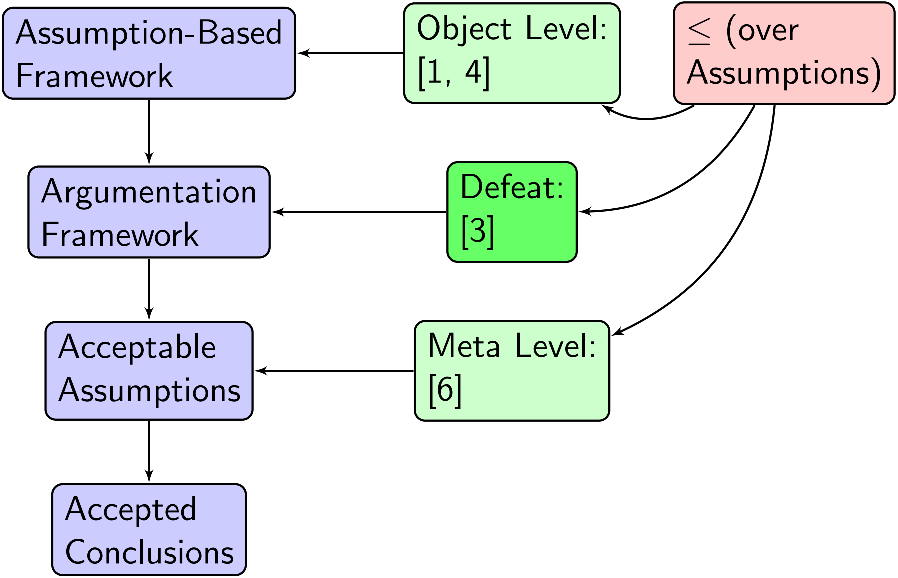
Definition (Attacks)

Given an assumption-based framework $\texttt{ABF} = (\mathcal{L}, \mathcal{R}, Ab, \overline{}, v)$, *a set of assumptions* $\Delta \subseteq Ab$:

- Δ *attacks an assumption* $A \in Ab$ *iff* $\Delta' \vdash_{\mathcal{R}} \overline{A}$ *for some* $\Delta' \subseteq \Delta$.
- Δ *attacks a set of assumptions* $\Theta \subseteq Ab$ *iff* Δ *attacks some* $A \in \Theta$.

We will also write $\Delta \hookrightarrow_f \Theta$ *if* Δ $\texttt{f}$*-defeats* Θ.

The Argumentation Pipeline: where do Priorities come in?



Comparing Sets of Assumptions

Definition (Lifting $\leq$)

Given an assumption-based framework $\mathtt{ABF} = (\mathcal{L}, \mathcal{R}, Ab, \overline{}, v)$ *and* $\Delta \subseteq Ab$*, we define:*

- $\emptyset \not< A$ *for any* $A \in Ab$ *and*
- $\Delta < A$ *if* $v(B) < v(A)$ *where* $\{B\} = \min(\Delta)$.

From Attack to Defeat

Definition (Attack, defeat, reverse defeat)

Given an assumption-based framework $\texttt{ABF} = (\mathcal{L}, \mathcal{R}, Ab, \overline{}, v)$ *is a set of assumptions* $\Delta \subseteq Ab$ *and an assumption* $A \in Ab$*, we say that:*

- Δ $\texttt{d}$-*defeats* A *iff there is a* $\Delta' \subseteq \Delta$ *s.t.* $\Delta' \vdash_{\mathcal{R}} \overline{A}$ *and* $\Delta' \not< A$.
- Δ $\texttt{d}$-*defeats* Θ *if* Δ $\texttt{d}$-*defeats some* $A \in \Theta$.
- Δ $\texttt{r}$-*defeats* $\Theta \subseteq Ab$ *iff either*
  - Δ $\texttt{d}$-*defeats* Θ, *or*
  - *there is a* $\Theta' \subseteq \Theta$ *s.t.* $\Theta' \vdash_{\mathcal{R}} \overline{A}$, $A \in \Delta$ *and* $A > \Theta'$

We will also denote $\texttt{d}$-defeat and $\texttt{r}$-defeat with, respectively, the symbols $\hookrightarrow_d$ and $\hookrightarrow_r$.

Example

- Björn wants to go out with his friends Agnetha (A), Benny (B) and Frida (F).
- If Benny is together with Agnetha, he doesn't want to go out with Frida ($A, B \rightarrow \overline{F}$).
- Björn likes Benny more then Agnetha ($v(A) = 1$ and $v(B) = 2$).
- Björn likes Frida more then Benny ($v(F) = 3$).

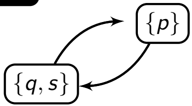
$$\{A, B\} \hookrightarrow_f \{F\}$$
$$\{F\} \hookrightarrow_r \{A, B\} \quad \{A, B\} \not\hookrightarrow_d \{F\}$$

Conflict-Free Sets of Assumptions

Example (for f-defeat)

- $Ab = \{p, q, s\}$.
- $\mathcal{R} = \{q \rightarrow \overline{p}; p \rightarrow \overline{q}\}$
- $\{q\} \vdash_{\mathcal{R}} \overline{p}$.
- $\{p\} \vdash_{\mathcal{R}} \overline{q}$.
- $\{q\} \vdash_{\mathcal{R}} s$.

Extensions :



Definition (Argumentation semantics)

Where $\Delta \subseteq Ab$ and $x \in \{d, r, f\}$, Δ is:

Conflict-Free Sets of Assumptions

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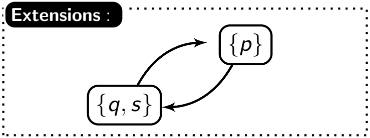

Definition (Argumentation semantics)

Where $\Delta \subseteq Ab$ and $x \in \{d, r, f\}$, Δ is:

- x-**conflict-free** *iff for every* $\Delta' \cup \Delta'' \subseteq \Delta$, $\Delta' \not\rightarrowtail_x \Delta''$.

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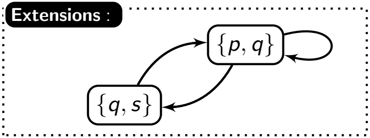

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Admissibility Semantics

Example

- $Ab = \{p, q, s\}$.
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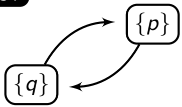
- $\{p\} \vdash_{\mathcal{R}} \overline{q}$.

Extensions :



Definition (Argumentation semantics)

Where $\Delta \subseteq Ab$ and $x \in \{d, r, f\}$, Δ is:

- *is x-**admissible** iff it is x-conflict-free and for each set of assumptions $\Theta \subseteq Ab$, if $\Theta \hookrightarrow_x \Delta$, then $\Delta \hookrightarrow_x \Theta$.*

Admissibility Semantics

Example

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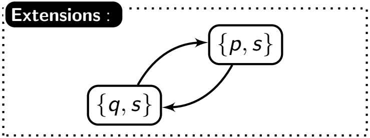

Definition (Argumentation semantics)

Where $\Delta \subseteq Ab$ and $x \in \{d, r, f\}$, Δ is:

- *is x-**admissible** iff it is x-conflict-free and for each set of assumptions $\Theta \subseteq Ab$, if $\Theta \hookrightarrow_x \Delta$, then $\Delta \hookrightarrow_x \Theta$.*
- *is x-**preferred** iff it is maximally (w.r.t. set inclusion) x-admissible.*

The Relation between d- and r-defeat

The Relation between d- and r-defeat

$$\{A, B\} \hookrightarrow_f \{F\}$$
$$\{F\} \hookrightarrow_r \{A, B\} \quad \{A, B\} \not\hookrightarrow_d \{F\}$$

Definition (Contraposition [5])

$\mathtt{ABF} = (\mathcal{L}, \mathcal{R}, Ab, \overline{}, v)$ *is closed under contraposition if for every* $\Delta \subseteq Ab$*:*

$$\textit{if } \Delta \vdash_{\mathcal{R}} \overline{A}$$

then for every $B \in \Delta$ *it holds that*

$$\{A\} \cup (\Delta \setminus \{B\}) \vdash_{\mathcal{R}} \overline{B}.$$

The Relation between $\mathtt{d}$- and $\mathtt{r}$-defeat

$$\{A, B\} \hookrightarrow_f \{F\}$$
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Definition (Contraposition [5])

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Conjecture

- $\mathtt{r}$-defeat seems to be a kind of contraposition.
- So perhaps if $\mathtt{ABF}$ is closed under contraposition, $\mathtt{r}$-defeat and $\mathtt{d}$-defeat coincide?

Well. . . [5]

Let $Ab = \{p, q, r, s\}$ and $v(s) = 1$, $v(p) = v(q) = 2$ and $v(r) = 3$ and

$$\mathcal{R} = \left\{\begin{matrix} p, q \to \overline{r} & p, r \to \overline{q} & q, r \to \overline{p} \\ p, q \to \overline{s} & p, s \to \overline{q} & q, s \to \overline{p} \\ & p \to \overline{p} \quad q \to \overline{q} & \end{matrix}\right\}$$

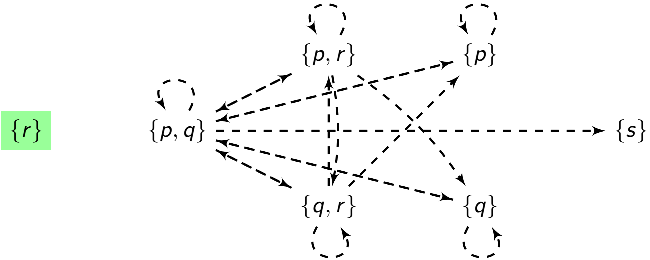

Figure: d-defeats are represented by dashed arrow whereas r-defeats are represented by dotted-arrows.

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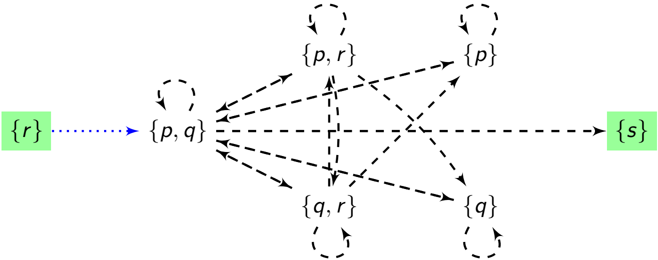

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Let $Ab = \{p, q, r, s\}$ and $v(s) = 1$, $v(p) = v(q) = 2$ and $v(r) = 3$ and

$$\mathcal{R} = \left\{ \begin{array}{ccc} p, q \to \overline{r} & p, r \to \overline{q} & q, r \to \overline{p} \\ p, q \to \overline{s} & p, s \to \overline{q} & q, s \to \overline{p} \\ \multicolumn{3}{c}{p \to \overline{p} \qquad q \to \overline{q}} \end{array} \right\}$$

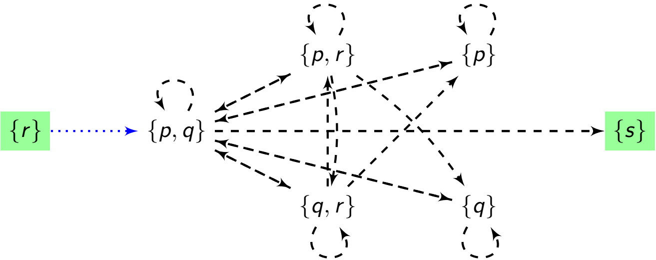

Figure: d-defeats are represented by dashed arrow whereas r-defeats are represented by dotted-arrows.

Note the large amount of self-defeating sets of assumptions.

The Relation between d- and r-defeat

Definition (Cycle-Freeness)

$\texttt{ABF} = (\mathcal{L}, \mathcal{R}, Ab, \overline{}, v)$ *is cycle-free if for every* $\Delta \subseteq Ab$*: if* $A \in \Delta$ *then:*

$$\Delta \not\vdash_{\mathcal{R}} \overline{A}.$$

Theorem

If ABF *is closed under contraposition and cycle-free then:*
Δ *is* d*-preferred iff* Δ *is* r*-preferred.*

Cycle-Free ABFs

- Cycle-Free ABFs have not been studied in the literature yet.
- Seems a valuable concept (e.g. for studying crash-resistance in ABA).

Results

		contraposition	well-behaved
d-cf(ABF) ⊆ r-cf(ABF)	Ex. 4	Thm. 6	Thm. 6
r-cf(ABF) ⊆ d-cf(ABF)	Fact 3	Fact 3	Fact 3
d-adm(ABF) ⊆ r-adm(ABF)	Ex. 4	Thm. 6	Thm. 6
r-adm(ABF) ⊆ d-adm(ABF)	Ex. 5	Ex. 5	Ex. 5
d-pref(ABF) ⊆ r-pref(ABF)	Ex. 4	Ex. 6	Thm. 7
r-pref(ABF) ⊆ d-pref(ABF)	Ex. 4	Ex. 6	Thm. 7
every d-comp(ABF) is subset of some r-comp(ABF)	Ex. 4	Thm. 6	Thm. 6
r-comp(ABF) ⊆ d-comp(ABF)	Ex. 4	Ex. 5	Ex. 5
d-stab(ABF) ⊆ r-stab(ABF)	Ex. 4	Thm. 6	Thm. 6
r-stab(ABF) ⊆ d-stab(ABF)	Ex. 4	Thm. 6	Thm. 6
every d-grou(ABF) is subset of some r-grou(ABF)	Ex. 4	Thm. 6	Thm. 6
every r-grou(ABF) is subset of some d-grou(ABF)	Ex. 5	Ex. 5	Ex. 5

Properties for Non-Monotonic Reasoning

Definition

Where $\texttt{ABF} = (\mathcal{L}, \mathcal{R}, Ab, \overline{}, v)$, $A, B \in \mathcal{L}$, $\texttt{ABF}^A = (\mathcal{L}, \mathcal{R} \cup \{\to A\}, Ab \setminus \{A\}, \overline{}, v)$, $\mathsf{sem} \in \{\mathsf{grou}, \mathsf{pref}, \mathsf{stab}\}$, *and* $\texttt{x} \in \{\texttt{r}, \texttt{d}\}$, *we say that* $\texttt{ABF}$ *(relative to* $\mathrel{|\!\sim}^{\mathsf{sem}}_{\texttt{x}}$*) satisfies:*

- *Cautious Cut (CC) iff: if* $\texttt{ABF} \mathrel{|\!\sim}^{\mathsf{sem}}_{\texttt{x}} A$ *and* $\texttt{ABF}^A \mathrel{|\!\sim}^{\mathsf{sem}}_{\texttt{x}} B$ *then* $\texttt{ABF} \mathrel{|\!\sim}^{\mathsf{sem}}_{\texttt{x}} B$
- *Cautious Monotony (CM) iff: if* $\texttt{ABF} \mathrel{|\!\sim}^{\mathsf{sem}}_{\texttt{x}} A$ *and* $\texttt{ABF} \mathrel{|\!\sim}^{\mathsf{sem}}_{\texttt{x}} B$ *then* $\texttt{ABF}^A \mathrel{|\!\sim}^{\mathsf{sem}}_{\texttt{x}} B$
- *Cumulativity iff it satisfies CC and CM relative to* $\mathrel{|\!\sim}^{\mathsf{sem}}_{\texttt{x}}$.

Results

		contr.	well-behaved
CC, d-grou	Ex. 9	Thm 9	Thm 9
CC, d-pref	Ex. 9	Thm 10	Thm 10
CC, d-stab	Ex. 9	Thm 10	Thm 10
CC, r-grou	Ex. 10	Ex. 14	Ex. 14
CC, r-pref	Ex. 10	Ex. 12	Thm 11
CC, r-stab	Thm 8	Thm 8	Thm 8

		contr.	well-behaved
CM, d-grou	Ex. 9	Thm 9	Thm 9
CM, d-pref	Ex. 11	Ex. 11	Thm 11
CM, d-stab	Ex. 11	Ex. 11	Thm 11
CM, r-grou	Ex. 13	Ex. 13	Ex. 15
CM, r-pref	Ex. 13	Ex. 13	Thm 11
CM, r-stab	Ex. 16	Ex. 16	Thm 11

Connection with Preferred Subtheories

Preferred Subtheories

Definition (Adapted from [2].)

Where $\mathtt{ABF} = (\mathcal{L}, \mathcal{R}, Ab, \overline{}, v)$*,*

- $\mathsf{IS}(\mathtt{ABF})$ *is the set of all* $\Delta \subseteq Ab$ *s.t.* $\Delta \setminus \{A\} \vdash_{\mathcal{R}} \overline{A}$ *for some* $A \in \Delta$*.*
- $\mathsf{CS}(\mathtt{ABF})$ *is the set of all* $\Delta \subseteq Ab$ *s.t. for no* $\Theta \in \mathsf{IS}(\mathtt{ABF})$, $\Theta \subseteq \Delta$*.*
- $\mathsf{MCS}(\mathtt{ABF})$ *is the set of all* $\Delta \in \mathsf{CS}(\mathtt{ABF})$ *that are maximal (w.r.t. set inclusion).*
- *Where* $\Delta \subseteq Ab$ *and* $i \in \mathbb{N}$, $\pi_i(\Delta) = \{A \in \Delta \mid v(A) = i\}$*.*
- $\prec \subseteq \wp(Ab) \times \wp(Ab)$ *is defined as:* $\Delta \prec \Theta$ *iff there is an* $i \geq 1$ *s.t.* $\pi_j(\Delta) = \pi_j(\Theta)$ *for every* $j > i$ *and* $\pi_i(\Delta) \subset \pi_i(\Theta)$*.*
- $\mathsf{MCS}_{\prec}(\mathtt{ABF}) = \max_{\prec}(\mathsf{MCS}(\mathtt{ABF}))$[a]

[a] Since we assume Ab to be finite, $\max_{\prec}(\mathsf{MCS}(\mathtt{ABF}))$ will never be empty.

Example

Let $Ab = \{p, q, r\}$ and $\mathcal{R} = \{p \to \overline{q}; q \to \overline{p}\}$ and $v(r) = 3$, $v(p) = 2$ and $v(q) = 1$.

- $\mathsf{IS}(\mathtt{ABF}) = \{\{p, q\}\}$.
- $\mathsf{MCS}(\mathtt{ABF}) = \{\{p, r\}, \{q, r\}\}$.

Now we compare the members of MCS lexicographically:

	$\{p, r\}$	$\{q, r\}$
3	r	r

Example

Let $Ab = \{p, q, r\}$ and $\mathcal{R} = \{p \to \overline{q}; q \to \overline{p}\}$ and $v(r) = 3$, $v(p) = 2$ and $v(q) = 1$.

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	$\{p, r\}$	$\{q, r\}$
3	r	r
2	p	

Example

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Now we compare the members of MCS lexicographically:

	$\{p, r\}$	$\{q, r\}$
3	r	r
2	p	
1		q

Representational Result.

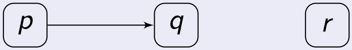
Theorem

For any well-behaved $\mathtt{ABF}$ *we have:*

$$\mathsf{MCS}_{\prec}(\mathtt{ABF}) = \mathtt{r}\text{-pref}(\mathtt{ABF}) = \mathtt{d}\text{-pref}(\mathtt{ABF}) = \mathtt{r}\text{-stab}(\mathtt{ABF}) = \mathtt{d}\text{-stab}(\mathtt{ABF})$$

Example

Let $Ab = \{p, q, r\}$ *and* $\mathcal{R} = \{p \rightarrow \overline{q}; q \rightarrow \overline{p}\}$ *and* $v(r) = 3$, $v(p) = 2$ *and* $v(q) = 1$.



In the paper

We also consider:

- The consistency postulate, and
- Dung's Fundamental Lemma.

Future Work

- ✓ Rational monotonicity.
- ✓ Translations between $\mathrm{ABA}^{\mathtt{d}}$, $\mathrm{ABA}^{\mathtt{r}}$ and $\mathrm{ABA}^{\mathtt{f}}$.
- × Partial orders.
- × Non-Flat Frameworks.
- × Prioritized logic programming.

Thank you!
Questions or remarks?

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