偶然、无知和随附性

范杰

偶然、无知和随附性是三个重要的哲学概念。本报告先回顾偶然、无知这两个概念的逻辑研究历史，在此基础上介绍本人最近在做的几个工作。如果还有时间，本人还将介绍与偶然逻辑相关的一点研究——随附性的模态逻辑。

A Constructive Four-Valued Logic

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1 Introduction

The Belnap-Dunn four-valued logic is the logic of De Morgan lattices. A De Morgan lattice is an algebra $(A, \wedge, \vee, \neg, 0, 1)$ where $(A, \wedge, \vee, 0, 1)$ is a bounded distributive lattice and $\neg$ is the De Morgan negation, namely $\neg$ is an unary operation on A satisfying the following conditions:

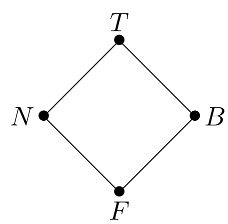
(1) $\neg(a \wedge b) = \neg a \vee \neg b$;

(2) $\neg(a \vee b) = \neg a \wedge \neg b$;

(3) $\neg\neg a = a$;

(4) $\neg 0 = 1$ and $\neg 1 = 0$.

It is well-known that every De Morgan lattice can be embedded in a (subdirect) product of the lattice **4**. Belnap's four-valued logic [1] is the logic of the following lattice **4**:



Belnap's Lattice **4**

Dunn's logic of De Morgan lattices [4, 5] is the same as Belnap's four-valued logic due to the representation theorem. Dunn [2] developed a theory of negation which is adapted with the theory of information. For a comprehensive survey on negation, see Dunn [4].

In the present paper, we shall present a constructive four-valued logic **C4L**. Dunn's four-valued semantics for De Morgan logic introduces two semantics consequence relations $\varphi \models_1 \psi$ and $\varphi \models_0 \psi$ which can be interpreted via Belnap's concepts of acceptance and rejection. A formula φ is accepted if 1 belongs to the value of φ, and it is rejected if 0 belongs to the value of φ. Our idea is to generalize Belnap-Dunn four-valued logic to a weak logic which is constructive in the following sense: if a formula is accepted, then it is accepted at any future state, and if it is rejected, it is rejected at any future state. The underling temporal structure is assumed to be a linear order. We call this logic as a constructive four-valued logic because it is a sublogic of Belnap-Dunn four-valued logic in which the law of double negation elimination is refuted. The logic **C4L** can be viewed the weakening of Belnap-Dunn logic in the way that intuitionistic logic is the weakening of classical propositional logic. We shall introduce the Kripke semantics for **C4L**. Consequently, Belnap-Dunn four-valued logic can be represented as the logic of a single reflexive point which is an extension of **C4L**.

The negation in **C4L** is a new one to Dunn's kite of negations [4]. Intuitively it is a modal negation which is interpreted on linearly ordered sets.

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STRONG FINITE MODEL PROPERTY OF PRE-ROUGH ALGEBRAS

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Various abstract algebraic structures have been studied since the inception of rough set theory in 1982 ([3]). One important structure is called pre-rough algebra which was first defined in [1] and developed in [2, 4, 5]. Predecessors of pre-rough algebras are topological Boolean algebras (tqBa) [6] and tqBa5 and MD'S5 which are defined and studied in [4, 5]. In the present paper, we prove the strong finite model property of pre-rough algebras and its predecessors tqBa, tqBa5 and MD'S5 by the method from substructural logics [7]. Further we present a decision procedure for all these pre-rough logics. Our results give a positive answer to the decidability problem of pre-rough algebras and show the connection between Substructural and pre-rough logics.

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A new framework for epistemic logic

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Inspired by nonstandard epistemic logics of "knowing how", "knowing why", "knowing what" and so on, we lay out a new foundation for epistemic logic by introducing a modality combining an existential quantifier and a box modality together. The new framework is a well-behaved yet powerful fragment of first-order modal logic, which shares most of the nice properties of propositional modal logic.

On Modal Reduction Principles

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A modal reduction principle is a modal formula of the form $Mp \to Np$ where M and N are finite strings of [] or <>. Van Benthem [1976] proves that every modal reduction principle has first-order correspondent over the class of all transitive frames. Zakharyaschev [1997] proves that the normal modal logics $\mathbf{K4} \oplus Mp \to Np$ has the finite model property. One intriguing open problem in modal logic is as follows: Do the normal modal logics $\mathbf{K} \oplus []^n p \to []^m p$ have the finite model property? This problem has already been open since 1970s. In this talk, we shall summarize results of finite model property in modal logic, and then we introduce a method from the study of finite embeddability property of residuated groupoids, proposed by Farulewski [2008] and developed by Buszkowski & Farulewski [2009] and Lin [2011], to solve half of the open problem, namely, $\mathbf{K} \oplus []^n p \to []^m p$ for the case $m < n$ has the finite model property (and hence are decidable). Some more general results can be derived from the method. It turns out that the model theoretic aspect of these modal reduction principles may be achieved in terms of algebra plus proof theory.

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Lyndon interpolation of the instantial neighborhood logic

A constructive proof

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Instantial neighborhood logic (INL) generalizes neighborhood logic (NL) by a language extension. Instead of \Box\phi (the current state has a neighborhood in which \phi holds everywhere), INL has formulas like \Box(\phi_1,…,\phi_j;\phi_0) ('\Box\phi_0' holds as in NL, and moreover in the evidential neighborhood \phi_1,…,\phi_j each holds somewhere resp.). In this talk, we will present a calculus-based constructive proof of INL's Lyndon interpolation theorem.