Self-referentiality in the framework of justification logics

俞珺华 Yu, Junhua

Department of Philosophy, Tsinghua University

2017.04.14 @ Zhejiang University

イロト イポト イヨト イヨト



- Realization in Justification Logic
- Self-referentiality
- Properties of non-self-referential fragments

Justification Logics JI Realization

• Realization in Justification Logic

ヘロト 人間 とくほとくほとう

∃ 𝒫𝔄𝔄

Justification Logics JL Realization

Justification logics JL

• Explicit versions of modal logics ML.

- $\Box \phi$ v.s. $t : \phi$,
- *t* explains contents implicitly indicated by \Box .
- Language: propositional, extended by *t*: *φ*.
 - *t* is a term (inductively defined, sensitive to logics),
 - ϕ is a formula in **this** language (where terms may occur in).
- The family of JL: >30 members, serving as explicit versions to many well-known ML's.
 - We will focus on the following five pairs:

ML K D T K4 S4

JL J JD JT J4 LP

・ロト ・ 同ト ・ ヨト ・ ヨト … ヨ

Justification Logics JL Realization

Justification logics JL

• Explicit versions of modal logics ML.

- □φ v.s. t:φ,
- *t* explains contents implicitly indicated by \Box .
- Language: propositional, extended by *t*: *φ*.
 - *t* is a term (inductively defined, sensitive to logics),
 - ϕ is a formula in this language (where terms may occur in).
- The family of JL: >30 members, serving as explicit versions to many well-known ML's.
 - We will focus on the following five pairs:

ML K D T K4 S4

JL J JD JT J4 LP

・ 同 ト ・ 国 ト ・ 国 ト …

Justification Logics JL Realization

The Logic of Proofs LP as an example

- By Artemov in 1995.
- $\phi := \perp |\rho| \phi \rightarrow \phi |t; \phi,$ $t := c |x| t \cdot t |t+t|!t.$
- Axiom schemes:
 - Classical propositional axioms,
 - $t: \phi \rightarrow \phi$,
 - $t_1: (\phi \rightarrow \psi) \rightarrow (t_2: \phi \rightarrow t_1 \cdot t_2: \psi),$
 - $t: \phi \rightarrow !t: t: \phi$,
 - $t_1: \phi \rightarrow t_1 + t_2: \phi$ and $t_2: \phi \rightarrow t_1 + t_2: \phi$.
- Rules schemes:

- $\vdash c : A$, where c is a constant, and A is an axiom.
- Explicit version of modal logic S4.
- Formally, the implicit/explicit correspondence is called realization.

Justification Logics JL Realization

The Logic of Proofs LP as an example

- By Artemov in 1995.
- $\phi := \bot | \boldsymbol{p} | \phi \rightarrow \phi | t : \phi,$
 - $t := c |x| t \cdot t |t+t|!t.$
- Axiom schemes:
 - Classical propositional axioms,
 - $t: \phi \rightarrow \phi$,
 - $t_1: (\phi \rightarrow \psi) \rightarrow (t_2: \phi \rightarrow t_1 \cdot t_2: \psi),$
 - $t: \phi \rightarrow !t: t: \phi$,
 - $t_1: \phi \to t_1 + t_2: \phi \text{ and } t_2: \phi \to t_1 + t_2: \phi$.
- Rules schemes:

- $\vdash c : A$, where c is a constant, and A is an axiom.
- Explicit version of modal logic S4.
- Formally, the implicit/explicit correspondence is called realization.

Justification Logics JL Realization

The Logic of Proofs LP as an example

- By Artemov in 1995.
- $\phi := \bot | \boldsymbol{p} | \phi \rightarrow \phi | t : \phi,$
 - $t := c |x| t \cdot t |t+t|!t.$
- Axiom schemes:
 - Classical propositional axioms,
 - $t: \phi \rightarrow \phi$,
 - $t_1: (\phi \rightarrow \psi) \rightarrow (t_2: \phi \rightarrow t_1 \cdot t_2: \psi),$
 - $t: \phi \rightarrow !t: t: \phi$,
 - $t_1: \phi \to t_1 + t_2: \phi \text{ and } t_2: \phi \to t_1 + t_2: \phi$.
- Rules schemes:

- $\vdash c : A$, where c is a constant, and A is an axiom.
- Explicit version of modal logic S4.
- Formally, the implicit/explicit correspondence is called realization.

Justification Logics JL Realization

The Logic of Proofs LP as an example

- By Artemov in 1995.
- $\phi := \bot | \boldsymbol{p} | \phi \rightarrow \phi | t : \phi,$
 - $t := c |x| t \cdot t |t+t|!t.$
- Axiom schemes:
 - Classical propositional axioms,
 - $t: \phi \rightarrow \phi$,
 - $t_1: (\phi \rightarrow \psi) \rightarrow (t_2: \phi \rightarrow t_1 \cdot t_2: \psi),$
 - $t: \phi \rightarrow !t: t: \phi$,
 - $t_1: \phi \to t_1 + t_2: \phi \text{ and } t_2: \phi \to t_1 + t_2: \phi$.
- Rules schemes:

- $\vdash c : A$, where c is a constant, and A is an axiom.
- Explicit version of modal logic S4.
- Formally, the implicit/explicit correspondence is called realization.

Justification Logics JL Realization

Realization

- Realizer
 - A mapping: the language of ML ~> that of a JL;
 - Assigns a term to each \Box -occurrence in the input formula.
- Realization
 - Given realizer (·)^r and modal formula φ, the image φ^r is a potential realization;
 - ϕ^r is a realization if further $JL \vdash \phi^r$.

イロト 不得 とくほ とくほとう

Realization

- Realizer
 - A mapping: the language of ML ~> that of a JL;
 - Assigns a term to each \Box -occurrence in the input formula.

Realization

- Realization
 - Given realizer (·)^r and modal formula φ, the image φ^r is a potential realization;
 - ϕ^r is a realization if further $JL \vdash \phi^r$.

イロト イポト イヨト イヨト

Justification Logics JL Realization

Realization (continued)

- Realization theorem (Artemov 1995 & Brezhnev 2000)
 - For any modal formula ϕ :
 - Let $X \in \{K, D, T, K4, S4\}$,
 - and $Y \in \{J, JD, JT, J4, LP\},$ resp.,
 - Then what follows are equivalent:
 - $X \vdash \phi$;
 - $\mathbf{Y} \vdash \phi^r$ for some realizer $(\cdot)^r$.

・ 同 ト ・ ヨ ト ・ ヨ ト

Realization in JL	In Justification Logics
Self-referentiality	In Modal Logics
Properties of NR Fragments	In Intuitionistic Propositional Logic

Self-referentiality

イロン イロン イヨン イヨン

In Justification Logics In Modal Logics In Intuitionistic Propositional Logic

Self-referential JL-formulas

• (recalled) Justification language (LP as an example)

- Formula $\phi := \bot | p | \phi \rightarrow \phi | t : \phi;$
- Term $t := c | x | t \cdot t | t + t | !t$.
- self-referential formulas like $t: \phi(t)$

• even c: A(c) is possible.

イロト イポト イヨト イヨト

In Justification Logics In Modal Logics In Intuitionistic Propositional Logic

Self-referential JL-formulas

- (recalled) Justification language (LP as an example)
 - Formula $\phi := \bot | p | \phi \rightarrow \phi | t : \phi;$
 - Term $t := c |x| t \cdot t |t+t|!t$.
- self-referential formulas like $t: \phi(t)$
 - even c: A(c) is possible.

イロト イ理ト イヨト イヨト

In Justification Logics In Modal Logics In Intuitionistic Propositional Logic

Constant specification \mathcal{CS}

- Definition (take LP as our example):
 - A set of formulas of the form *c*: *A*.
- Link axioms with constants that present them in terms.
- JL(*CS*) is the fragment of JL where rule scheme *AN* can only put formulas from *CS*.
 - e.g., $JL(\emptyset)$ is the fragment of JL without AN.

In Justification Logics In Modal Logics In Intuitionistic Propositional Logic

Constant specification \mathcal{CS}

- Definition (take LP as our example):
 - A set of formulas of the form *c*: *A*.
- Link axioms with constants that present them in terms.
- JL(CS) is the fragment of JL where rule scheme AN can only put formulas from CS.
 - e.g., $JL(\emptyset)$ is the fragment of JL without AN.

In Justification Logics In Modal Logics In Intuitionistic Propositional Logic

Self-referentiality of \mathcal{CS}

- Take LP as our example.
- CS is (directly) self-referential, if for some c and A

 $c: A(c) \in CS.$

- Let $CS^* := \{c: A \mid c \text{ does not occur in } A\};$
 - The largest non-self-referential constant specification.
 - Thus, JL(*CS*^{*}) is the fragment of JL where *AN* can only introduce non-self-referential formulas.

In Justification Logics In Modal Logics In Intuitionistic Propositional Logic

Self-referentiality of \mathcal{CS}

- Take LP as our example.
- CS is (directly) self-referential, if for some c and A

$$c: A(c) \in CS.$$

- Let $CS^* := \{c: A \mid c \text{ does not occur in } A\};$
 - The largest non-self-referential constant specification.
 - Thus, JL(CS^{*}) is the fragment of JL where AN can only introduce non-self-referential formulas.

ML^{NR}: non-self-referential realizable fragment of ML

• Definition:

- Let $X \in \{K, D, T, K4, S4\}$, and $Y \in \{J, JD, JT, J4, LP\}$, resp.;
- $X^{NR} := \{X \vdash \phi \mid Y(\mathcal{CS}^*) \vdash \phi^r \text{ for some realizer } (\cdot)^r\}.$
- A model theorem is non-self-referential if being in ML^{NR}, and self-referential otherwise.
- Self-referential modal-theorems exist. (Kuznets 2006 & 2008):
 - $K^{NR} = K$
 - $\mathsf{D}^{NR} = \mathsf{D}$
 - $\Diamond(p \to \Box p) \in \mathsf{T} \setminus \mathsf{T}^{NR}$
 - $\Box \neg (p \rightarrow \Box p) \rightarrow \Box \bot \in \mathsf{K4} \setminus \mathsf{K4}^{\mathsf{NI}}$
 - $\bullet \hspace{0.1 cm} \Diamond(p \rightarrow \Box p) \in \mathsf{S4} \setminus \mathsf{S4}^{\mathsf{NR}}$

ヘロト ヘアト ヘビト ヘビト

3

ML^{NR}: non-self-referential realizable fragment of ML

• Definition:

- Let $X \in \{K, D, T, K4, S4\},$ and $Y \in \{J, JD, JT, J4, LP\},$ resp.;
- $X^{NR} := \{X \vdash \phi \mid Y(\mathcal{CS}^*) \vdash \phi^r \text{ for some realizer } (\cdot)^r\}.$
- A model theorem is non-self-referential if being in ML^{NR}, and self-referential otherwise.
- Self-referential modal-theorems exist. (Kuznets 2006 & 2008):
 - $K^{NR} = K$
 - $D^{NR} = D$
 - $\Diamond(p \to \Box p) \in \mathsf{T} \setminus \mathsf{T}^{NR}$
 - $\Box \neg (p \rightarrow \Box p) \rightarrow \Box \bot \in \mathsf{K4} \setminus \mathsf{K4}^{\mathsf{NI}}$
 - $\bullet \hspace{0.1 cm} \Diamond(p \rightarrow \Box p) \in \mathsf{S4} \setminus \mathsf{S4}^{\textit{NR}}$

ヘロト ヘアト ヘビト ヘビト

3

ML^{NR}: non-self-referential realizable fragment of ML

• Definition:

- Let $X \in \{K, D, T, K4, S4\},$ and $Y \in \{J, JD, JT, J4, LP\},$ resp.;
- $X^{NR} := \{X \vdash \phi \mid Y(\mathcal{CS}^*) \vdash \phi^r \text{ for some realizer } (\cdot)^r\}.$
- A model theorem is non-self-referential if being in ML^{NR}, and self-referential otherwise.
- Self-referential modal-theorems exist. (Kuznets 2006 & 2008):
 - $K^{NR} = K$
 - $\mathsf{D}^{NR} = \mathsf{D}$
 - $\Diamond(p \to \Box p) \in \mathsf{T} \setminus \mathsf{T}^{NR}$
 - $\Box \neg (p \rightarrow \Box p) \rightarrow \Box \bot \in \mathsf{K4} \setminus \mathsf{K4}^{NR}$
 - $\Diamond(p \to \Box p) \in S4 \setminus S4^{NR}$

・ 同 ト ・ ヨ ト ・ ヨ ト

ML^{NR}: non-self-referential realizable fragment of ML

• Definition:

- Let $X \in \{K, D, T, K4, S4\},$ and $Y \in \{J, JD, JT, J4, LP\},$ resp.;
- $\mathsf{X}^{NR} := {\{\mathsf{X} \vdash \phi \mid \mathsf{Y}(\mathcal{CS}^*) \vdash \phi^r \text{ for some realizer } (\cdot)^r\}}.$
- A model theorem is non-self-referential if being in ML^{NR}, and self-referential otherwise.
- Self-referential modal-theorems exist. (Kuznets 2006 & 2008):
 - $K^{NR} = K$
 - $\mathsf{D}^{NR} = \mathsf{D}$
 - $\Diamond(p \rightarrow \Box p) \in \mathsf{T} \setminus \mathsf{T}^{NR}$
 - $\Box \neg (p \rightarrow \Box p) \rightarrow \Box \bot \in \mathsf{K4} \setminus \mathsf{K4}^{NR}$
 - $(p \rightarrow \Box p) \in S4 \setminus S4^{NR}$

・聞き ・ヨキ ・ヨト

Realizing intuitionistic propositional logic IPC via S4

- The initial motivation of Artemov's LP;
- The Gödel–Artemov formalization of BHK semantics;
- Gödel's modal embedding (·)[△] is a mapping from propositional language to propositional modal language that satisfies:

$$\begin{cases} p^{\triangle} = \Box p \\ \bot^{\triangle} = \Box \bot \\ (\phi \oplus \psi)^{\triangle} = \Box (\phi^{\triangle} \oplus \psi^{\triangle}) \text{ for } \oplus \in \{\land, \lor, \rightarrow\}. \end{cases}$$

 Sound-and-faithfully embeds IPC into S4, i.e., IPC ⊢ φ iff S4 ⊢ φ[△] (McKinsey & Tarski 1948).

Realizing intuitionistic propositional logic IPC via S4

- The initial motivation of Artemov's LP;
- The Gödel-Artemov formalization of BHK semantics;
- Gödel's modal embedding (·)[△] is a mapping from propositional language to propositional modal language that satisfies:

$$\begin{cases} \boldsymbol{p}^{\triangle} = \Box \boldsymbol{p} \\ \bot^{\triangle} = \Box \bot \\ (\phi \oplus \psi)^{\triangle} = \Box (\phi^{\triangle} \oplus \psi^{\triangle}) \text{ for } \oplus \in \{\land, \lor, \rightarrow\}. \end{cases}$$

• Sound-and-faithfully embeds IPC into S4, i.e., IPC $\vdash \phi$ iff S4 $\vdash \phi^{\triangle}$ (McKinsey & Tarski 1948).

Realizing intuitionistic propositional logic IPC via S4

- The initial motivation of Artemov's LP;
- The Gödel-Artemov formalization of BHK semantics;
- Gödel's modal embedding (·)[△] is a mapping from propositional language to propositional modal language that satisfies:

$$\begin{cases} \boldsymbol{p}^{\bigtriangleup} = \Box \boldsymbol{p} \\ \bot^{\bigtriangleup} = \Box \bot \\ (\phi \oplus \psi)^{\bigtriangleup} = \Box (\phi^{\bigtriangleup} \oplus \psi^{\bigtriangleup}) \text{ for } \oplus \in \{\land, \lor, \rightarrow\}. \end{cases}$$

• Sound-and-faithfully embeds IPC into S4, i.e., IPC $\vdash \phi$ iff S4 $\vdash \phi^{\triangle}$ (McKinsey & Tarski 1948).

ヘロト ヘワト ヘビト ヘビト

In Justification Logics In Modal Logics In Intuitionistic Propositional Logic

Basic embeddings

• An extension of Gödel's modal embedding.

• A potential embedding $((\cdot)^{\times})$ is basic if (let $\odot \in \{\land,\lor\}$):

$$\begin{pmatrix} \phi^{\times} = \phi^{\times}_{+} \\ p^{\times}_{+} = \Box^{h_{+}}p \quad p^{\times}_{-} = \Box^{h_{-}}p \quad \text{similar for } \bot \\ (\phi \odot \psi)^{\times}_{+} = \Box^{j_{\odot+}}(\Box^{k_{\odot+}}\phi^{\times}_{+} \odot \Box^{l_{\odot+}}\psi^{\times}_{+}) \\ (\phi \odot \psi)^{\times}_{-} = \Box^{j_{\odot-}}(\Box^{k_{\odot-}}\phi^{\times}_{-} \odot \Box^{l_{\odot-}}\psi^{\times}_{-}) \\ (\phi \rightarrow \psi)^{\times}_{+} = \Box^{j_{\rightarrow+}}(\Box^{k_{\rightarrow+}}\phi^{\times}_{+} \rightarrow \Box^{L_{\rightarrow+}}\psi^{\times}_{+}) \\ (\phi \rightarrow \psi)^{\times}_{-} = \Box^{j_{\rightarrow-}}(\Box^{k_{\rightarrow-}}\phi^{\times}_{+} \rightarrow \Box^{L_{\rightarrow-}}\psi^{\times}_{-})$$

- A basic embedding is a potential one that satisfies: IPC ⊢ φ iff S4 ⊢ φ[×].
 - possible applications on other logic pairs.

ヘロト ヘアト ヘビト ヘビト

1

In Justification Logics In Modal Logics In Intuitionistic Propositional Logic

Basic embeddings

- An extension of Gödel's modal embedding.
- A potential embedding $((\cdot)^{\times})$ is basic if (let $\odot \in \{\land,\lor\}$):

$$\left\{ \begin{array}{l} \phi^{\times} = \phi^{\times}_{+} \\ p^{\times}_{+} = \Box^{h_{+}} p \quad p^{\times}_{-} = \Box^{h_{-}} p \quad \text{similar for } \bot \\ (\phi \odot \psi)^{\times}_{+} = \Box^{j_{\odot +}} (\Box^{k_{\odot +}} \phi^{\times}_{+} \odot \Box^{l_{\odot +}} \psi^{\times}_{+}) \\ (\phi \odot \psi)^{\times}_{-} = \Box^{j_{\odot -}} (\Box^{k_{\odot -}} \phi^{\times}_{-} \odot \Box^{l_{\odot -}} \psi^{\times}_{-}) \\ (\phi \rightarrow \psi)^{\times}_{+} = \Box^{j_{\rightarrow +}} (\Box^{k_{\rightarrow +}} \phi^{\times}_{+} \rightarrow \Box^{L_{\rightarrow +}} \psi^{\times}_{+}) \\ (\phi \rightarrow \psi)^{\times}_{-} = \Box^{j_{\rightarrow -}} (\Box^{k_{\rightarrow -}} \phi^{\times}_{+} \rightarrow \Box^{L_{\rightarrow -}} \psi^{\times}_{-}) \end{array} \right.$$

- A basic embedding is a potential one that satisfies: IPC ⊢ φ iff S4 ⊢ φ[×].
 - possible applications on other logic pairs.

In Justification Logics In Modal Logics In Intuitionistic Propositional Logic

Basic embeddings

- An extension of Gödel's modal embedding.
- A potential embedding $((\cdot)^{\times})$ is basic if (let $\odot \in \{\land,\lor\}$):

$$\begin{cases} \phi^{\times} = \phi^{\times}_{+} \\ p^{\times}_{+} = \Box^{h_{+}} p \quad p^{\times}_{-} = \Box^{h_{-}} p \quad \text{similar for } \bot \\ (\phi \odot \psi)^{\times}_{+} = \Box^{j_{\odot^{+}}} (\Box^{k_{\odot^{+}}} \phi^{\times}_{+} \odot \Box^{l_{\odot^{+}}} \psi^{\times}_{+}) \\ (\phi \odot \psi)^{\times}_{-} = \Box^{j_{\odot^{-}}} (\Box^{k_{\odot^{-}}} \phi^{\times}_{-} \odot \Box^{l_{\odot^{-}}} \psi^{\times}_{-}) \\ (\phi \rightarrow \psi)^{\times}_{+} = \Box^{j_{\rightarrow^{+}}} (\Box^{k_{\rightarrow^{+}}} \phi^{\times}_{+} \rightarrow \Box^{L_{\rightarrow^{+}}} \psi^{\times}_{+}) \\ (\phi \rightarrow \psi)^{\times}_{-} = \Box^{j_{\rightarrow^{-}}} (\Box^{k_{\rightarrow^{-}}} \phi^{\times}_{+} \rightarrow \Box^{L_{\rightarrow^{-}}} \psi^{\times}_{-}) \end{cases}$$

 A basic embedding is a potential one that satisfies: IPC ⊢ φ iff S4 ⊢ φ[×].

• possible applications on other logic pairs.

ヘロト 人間 ト ヘヨト ヘヨト

IPC^{NR}: non-self-referential realizable fragment of IPC

• Definition:

- $\mathsf{IPC}^{\mathsf{NR}(\times)} := \{\mathsf{IPC} \vdash \phi \,|\, \phi^{\times} \in \mathsf{S4}^{\mathsf{NR}}\};$
- $\operatorname{IPC}^{NR} := \bigcup_{\times} \operatorname{IPC}^{NR(\times)};$
- An intuitionistic theorem is non-self-referential if being in IPC^{NR}, and self-referential otherwise;
- $\mathsf{IPC}_{\rightarrow}^{\mathit{NR}(\times)}$ and $\mathsf{IPC}_{\rightarrow}^{\mathit{NR}}$ are similarly defined based on $\mathsf{IPC}_{\rightarrow}.$
- Self-referential IPC-theorem exists. (Yu 2014):
 - $\{\neg \neg \alpha \mid \alpha \in \mathsf{CPC} \setminus \mathsf{IPC}\} \subseteq \mathsf{IPC} \setminus \mathsf{IPC}^{\mathsf{NR}}$
 - $((((p \rightarrow q) \rightarrow p) \rightarrow p) \rightarrow q) \rightarrow q \in \mathsf{IPC}_{\rightarrow} \setminus \mathsf{IPC}_{\rightarrow}^{NR}$

ヘロト ヘアト ヘビト ヘビト

IPC^{NR}: non-self-referential realizable fragment of IPC

Definition:

- $\mathsf{IPC}^{\mathsf{NR}(\times)} := \{\mathsf{IPC} \vdash \phi \mid \phi^{\times} \in \mathsf{S4}^{\mathsf{NR}}\};$
- $\operatorname{IPC}^{NR} := \bigcup_{\times} \operatorname{IPC}^{NR(\times)};$
- An intuitionistic theorem is non-self-referential if being in IPC^{NR}, and self-referential otherwise;
- $\mathsf{IPC}^{\mathit{NR}(\times)}_{\rightarrow}$ and $\mathsf{IPC}^{\mathit{NR}}_{\rightarrow}$ are similarly defined based on $\mathsf{IPC}_{\rightarrow}.$
- Self-referential IPC-theorem exists. (Yu 2014):
 - $\{\neg \neg \alpha \mid \alpha \in \mathsf{CPC} \setminus \mathsf{IPC}\} \subseteq \mathsf{IPC} \setminus \mathsf{IPC}^{\mathsf{NR}}$
 - $((((p \rightarrow q) \rightarrow p) \rightarrow p) \rightarrow q) \rightarrow q \in \mathsf{IPC}_{\rightarrow} \setminus \mathsf{IPC}_{\rightarrow}^{NR}$

ヘロト ヘアト ヘビト ヘビト

IPC^{NR}: non-self-referential realizable fragment of IPC

Definition:

- $\mathsf{IPC}^{\mathsf{NR}(\times)} := \{\mathsf{IPC} \vdash \phi \mid \phi^{\times} \in \mathsf{S4}^{\mathsf{NR}}\};$
- $IPC^{NR} := \bigcup_{\times} IPC^{NR(\times)};$
- An intuitionistic theorem is non-self-referential if being in IPC^{NR}, and self-referential otherwise;
- $IPC_{\rightarrow}^{NR(\times)}$ and IPC_{\rightarrow}^{NR} are similarly defined based on IPC_{\rightarrow} .
- Self-referential IPC-theorem exists. (Yu 2014):
 - $\{\neg \neg \alpha \mid \alpha \in \mathsf{CPC} \setminus \mathsf{IPC}\} \subseteq \mathsf{IPC} \setminus \mathsf{IPC}^{\mathsf{NR}}$
 - $\bullet \hspace{0.1 in} ((((p \!\rightarrow\! q) \!\rightarrow\! p) \!\rightarrow\! p) \!\rightarrow\! q) \!\rightarrow\! q \in \mathsf{IPC}_{\rightarrow} \setminus \mathsf{IPC}_{\rightarrow}^{\scriptscriptstyle NR}$

ヘロン 人間 とくほ とくほ とう

1

IPC^{NR}: non-self-referential realizable fragment of IPC

• Definition:

- $\operatorname{IPC}^{NR(\times)} := \{\operatorname{IPC} \vdash \phi \mid \phi^{\times} \in \operatorname{S4}^{NR}\};$
- $\operatorname{IPC}^{NR} := \bigcup_{\times} \operatorname{IPC}^{NR(\times)};$
- An intuitionistic theorem is non-self-referential if being in IPC^{NR}, and self-referential otherwise;
- $IPC_{\rightarrow}^{\textit{NR}(\times)}$ and $IPC_{\rightarrow}^{\textit{NR}}$ are similarly defined based on $IPC_{\rightarrow}.$
- Self-referential IPC-theorem exists. (Yu 2014):
 - $\{\neg \neg \alpha \mid \alpha \in \mathsf{CPC} \setminus \mathsf{IPC}\} \subseteq \mathsf{IPC} \setminus \mathsf{IPC}^{\mathsf{NR}}$
 - $((((p
 ightarrow q)
 ightarrow p)
 ightarrow q)
 ightarrow q \in \mathsf{IPC}_{
 ightarrow} \setminus \mathsf{IPC}_{
 ightarrow}^{NR}$

Realization in JL In Justification Lo Self-referentiality In Modal Logics roperties of NR Fragments In Intuitionistic Pr

In Modal Logics In Intuitionistic Propositional Logic

IPC^{NR}: non-self-referential realizable fragment of IPC

Definition:

- $\mathsf{IPC}^{\mathsf{NR}(\times)} := \{\mathsf{IPC} \vdash \phi \,|\, \phi^{\times} \in \mathsf{S4}^{\mathsf{NR}}\};$
- $\operatorname{IPC}^{NR} := \bigcup_{\times} \operatorname{IPC}^{NR(\times)};$
- An intuitionistic theorem is non-self-referential if being in IPC^{NR}, and self-referential otherwise;
- $\text{IPC}_{\rightarrow}^{\textit{NR}(\times)}$ and $\text{IPC}_{\rightarrow}^{\textit{NR}}$ are similarly defined based on $\text{IPC}_{\rightarrow}.$
- Self-referential IPC-theorem exists. (Yu 2014):
 - $\{\neg \neg \alpha \mid \alpha \in \mathsf{CPC} \setminus \mathsf{IPC}\} \subseteq \mathsf{IPC} \setminus \mathsf{IPC}^{\mathsf{NR}}$
 - $((((p \rightarrow q) \rightarrow p) \rightarrow p) \rightarrow q) \rightarrow q \in \mathsf{IPC}_{\rightarrow} \setminus \mathsf{IPC}_{\rightarrow}^{NR}$

ヘロン 人間 とくほ とくほ とう

-

IPC^{NR}: non-self-referential realizable fragment of IPC

• Definition:

- $\mathsf{IPC}^{\mathsf{NR}(\times)} := \{\mathsf{IPC} \vdash \phi \mid \phi^{\times} \in \mathsf{S4}^{\mathsf{NR}}\};$
- $\operatorname{IPC}^{NR} := \bigcup_{\times} \operatorname{IPC}^{NR(\times)};$
- An intuitionistic theorem is non-self-referential if being in IPC^{NR}, and self-referential otherwise;
- $\text{IPC}_{\rightarrow}^{\textit{NR}(\times)}$ and $\text{IPC}_{\rightarrow}^{\textit{NR}}$ are similarly defined based on $\text{IPC}_{\rightarrow}.$
- Self-referential IPC-theorem exists. (Yu 2014):
 - $\{\neg \neg \alpha \mid \alpha \in \mathsf{CPC} \setminus \mathsf{IPC}\} \subseteq \mathsf{IPC} \setminus \mathsf{IPC}^{\mathsf{NR}}$
 - $((((p \rightarrow q) \rightarrow p) \rightarrow p) \rightarrow q) \rightarrow q \in \mathsf{IPC}_{\rightarrow} \setminus \mathsf{IPC}_{\rightarrow}^{NR}$

Realization in JL	A Wieldy Tool
Self-referentiality	Failures of <i>MP</i>
Properties of NR Fragments	Between NR fragments of ML's

Properties of non-self-referential realizable fragments

<ロ> (四) (四) (日) (日) (日)

э
A Wieldy Tool Failures of *MP* Between *NR* fragments of ML's

Prehistoric-cycle-free provable fragment

- For each logic mentioned above,
 - the *CF* (prehistoric-cycle-free provable) fragment is a subset of

the NR (non-self-referential realizable) fragment;

- The best known approximation;
- Decidable, wieldy for simple formulas.

ヘロト ヘアト ヘビト ヘ

A Wieldy Tool Failures of *MP* Between *NR* fragments of ML's

The underline calculus G3[st4]

Ax.
$$\overline{\rho,\Gamma\Rightarrow\Delta,\rho}$$
 $L\perp$. $\overline{\perp,\Gamma\Rightarrow\Delta}$ $L\rightarrow$. $\Gamma\Rightarrow\Delta,\phi~\psi,\Gamma\Rightarrow\Delta$ $R\rightarrow$. $\overline{\psi,\Gamma\Rightarrow\Delta,\psi}$ $L\rightarrow$. $\overline{\psi,\Gamma\Rightarrow\Delta,\phi\to\psi}$ $R\rightarrow$. $\overline{\psi,\Gamma\Rightarrow\Delta,\psi\to\psi}$ $L\square$. $\frac{\theta,\Box\theta,\Gamma\Rightarrow\Delta}{\Box\theta,\Gamma\Rightarrow\Delta}$ $R\square$. $\frac{\Box\Theta\Rightarrow\eta}{\Box\Theta,\Gamma\Rightarrow\Delta,\Box\eta}$ $4\square$. $\frac{\Theta,\Box\Theta\Rightarrow\eta}{\Box\Theta,\Gamma\Rightarrow\Delta,\Box\eta}$ $K\square$. $\frac{\Theta\Rightarrow\eta}{\Box\Theta,\Gamma\Rightarrow\Delta,\Box\eta}$

- G3cp: $Ax, L \perp, L \rightarrow, R \rightarrow$;
- G3t: G3cp with $L\Box$, $K\Box$;
- $\Box \Theta := \{ \Box \theta \mid \theta \in \Theta \}.$

G3s: G3cp with $L\Box$, $R\Box$; G34: G3cp with $4\Box$.

ヘロト ヘアト ヘビト ヘビト

1

A Wieldy Tool Failures of *MP* Between *NR* fragments of ML's

Prehistoric graph and prehistoric cycle

- Given a proof tree $\mathcal{T} = (T, R)$, the prehistoric graph of \mathcal{T} is $\mathcal{P}(\mathcal{T}) := (F, \prec)$, where
 - *F* is the set of families of positive \Box 's in the proof tree T,

• (take G3s for instance)

$$\prec := \{ \langle i, j \rangle \mid \langle (\Box \Theta(\Box_i) \Rightarrow \eta), (\Box \Theta(\Box_i), \Gamma \Rightarrow \Delta, \Box_j \eta) \rangle \in R \},$$
• i.e., $\frac{\Box \Theta(\Box_i) \Rightarrow \eta}{\Box \Theta(\Box_i), \Gamma \Rightarrow \Delta, \Box_j \eta} (R \Box)$ is a step in \mathcal{T} .

- Given \mathcal{T} , a prehistoric cycle is a cycle in $\mathcal{P}(\mathcal{T})$.
- A proof \mathcal{T} is cycle-free, if $\mathcal{P}(\mathcal{T})$ has no cycle.

A Wieldy Tool Failures of *MP* Between *NR* fragments of ML's

Prehistoric graph and prehistoric cycle

- Given a proof tree $\mathcal{T} = (T, R)$, the prehistoric graph of \mathcal{T} is $\mathcal{P}(\mathcal{T}) := (F, \prec)$, where
 - *F* is the set of families of positive \Box 's in the proof tree T,

• (take G3s for instance)

$$\prec := \{ \langle i, j \rangle \mid \langle (\Box \Theta(\Box_i) \Rightarrow \eta), (\Box \Theta(\Box_i), \Gamma \Rightarrow \Delta, \Box_j \eta) \rangle \in R \},$$
• i.e., $\frac{\Box \Theta(\Box_i) \Rightarrow \eta}{\Box \Theta(\Box_i), \Gamma \Rightarrow \Delta, \Box_j \eta} (R \Box)$ is a step in \mathcal{T} .

- Given \mathcal{T} , a prehistoric cycle is a cycle in $\mathcal{P}(\mathcal{T})$.
- A proof \mathcal{T} is cycle-free, if $\mathcal{P}(\mathcal{T})$ has no cycle.

A Wieldy Tool Failures of *MP* Between *NR* fragments of ML's

Prehistoric graph and prehistoric cycle

- Given a proof tree $\mathcal{T} = (T, R)$, the prehistoric graph of \mathcal{T} is $\mathcal{P}(\mathcal{T}) := (F, \prec)$, where
 - *F* is the set of families of positive \Box 's in the proof tree T,

• (take G3s for instance)

$$\prec := \{ \langle i, j \rangle \mid \langle (\Box \Theta(\Box_i) \Rightarrow \eta), (\Box \Theta(\Box_i), \Gamma \Rightarrow \Delta, \Box_j \eta) \rangle \in R \},$$
• i.e., $\frac{\Box \Theta(\Box_i) \Rightarrow \eta}{\Box \Theta(\Box_i), \Gamma \Rightarrow \Delta, \Box_j \eta} (R \Box)$ is a step in \mathcal{T} .

- Given \mathcal{T} , a prehistoric cycle is a cycle in $\mathcal{P}(\mathcal{T})$.
- A proof \mathcal{T} is cycle-free, if $\mathcal{P}(\mathcal{T})$ has no cycle.

A Wieldy Tool Failures of *MP* Between *NR* fragments of ML's

Prehistoric graph and prehistoric cycle

- Given a proof tree $\mathcal{T} = (T, R)$, the prehistoric graph of \mathcal{T} is $\mathcal{P}(\mathcal{T}) := (F, \prec)$, where
 - *F* is the set of families of positive \Box 's in the proof tree T,

• (take G3s for instance)

$$\prec := \{ \langle i, j \rangle \mid \langle (\Box \Theta(\Box_i) \Rightarrow \eta), (\Box \Theta(\Box_i), \Gamma \Rightarrow \Delta, \Box_j \eta) \rangle \in R \},$$
• i.e., $\frac{\Box \Theta(\Box_i) \Rightarrow \eta}{\Box \Theta(\Box_i), \Gamma \Rightarrow \Delta, \Box_j \eta} (R \Box)$ is a step in \mathcal{T} .

- Given \mathcal{T} , a prehistoric cycle is a cycle in $\mathcal{P}(\mathcal{T})$.
- A proof \mathcal{T} is cycle-free, if $\mathcal{P}(\mathcal{T})$ has no cycle.

A Wieldy Tool Failures of *MP* Between *NR* fragments of ML's

Prehistoric-cycle-free fragments

Definition:

- Let $X \in \{T, K4, S4\}$, and $Y \in \{G3t, G34, G3s\}$, resp.;
 - $X^{CF} := \{\phi \mid (\Rightarrow \phi) \text{ has a cycle-free proof in } Y\}.$
- For a basic embedding $(\cdot)^{\times}$:
 - $\operatorname{IPC}_{\operatorname{CF}}^{\operatorname{CF}(\times)} := \{\operatorname{IPC} \vdash \phi \mid \phi^{\times} \in \operatorname{S4}^{\operatorname{CF}}\};$
 - $\mathsf{IPC}^{CF} := \bigcup_{\times} \mathsf{IPC}^{CF(\times)};$
 - $\bullet~ \mathsf{IPC}_{\rightarrow}^{\mathit{CF}(\times)}$ and $\mathsf{IPC}_{\rightarrow}^{\mathit{CF}}$ are similarly defined.
- $\in CF$ is sufficient to $\in NR$ (Yu 2010 & 2014):
 - If $X \in \{T, K4, S4, IPC, IPC_{\rightarrow}\}$, then $X^{CF} \subseteq X^{NR}$.

A Wieldy Tool Failures of *MP* Between *NR* fragments of ML's

Prehistoric-cycle-free fragments

Definition:

- Let $X \in \{T, K4, S4\},$ and $Y \in \{G3t, G34, G3s\},$ resp.;
 - $X^{CF} := \{\phi \mid (\Rightarrow \phi) \text{ has a cycle-free proof in } Y\}.$
- For a basic embedding $(\cdot)^{\times}$:
 - $\operatorname{IPC}_{\mathcal{F}}^{CF(\times)} := \{\operatorname{IPC} \vdash \phi \mid \phi^{\times} \in \operatorname{S4}^{CF}\};$
 - $\operatorname{IPC}^{CF} := \bigcup_{\times} \operatorname{IPC}^{CF(\times)};$
 - $\mathsf{IPC}_{\rightarrow}^{\mathit{CF}(\times)}$ and $\mathsf{IPC}_{\rightarrow}^{\mathit{CF}}$ are similarly defined.
- $\in CF$ is sufficient to $\in NR$ (Yu 2010 & 2014): • If $X \in \{T, K4, S4, IPC, IPC_{\rightarrow}\}$, then $X^{CF} \subseteq X^{NR}$.

A Wieldy Tool Failures of *MP* Between *NR* fragments of ML's

Prehistoric-cycle-free fragments

• Definition:

- Let $X \in \{T, K4, S4\},$ and $Y \in \{G3t, G34, G3s\},$ resp.;
 - $X^{CF} := \{\phi \mid (\Rightarrow \phi) \text{ has a cycle-free proof in } Y\}.$
- For a basic embedding $(\cdot)^{\times}$:
 - $\mathsf{IPC}^{CF(\times)} := \{\mathsf{IPC} \vdash \phi \mid \phi^{\times} \in \mathsf{S4}^{CF}\};$
 - $\mathsf{IPC}^{CF} := \bigcup_{\times} \mathsf{IPC}^{CF(\times)};$
 - $\mathsf{IPC}_{\rightarrow}^{\mathit{CF}(\times)}$ and $\mathsf{IPC}_{\rightarrow}^{\mathit{CF}}$ are similarly defined.
- $\in CF$ is sufficient to $\in NR$ (Yu 2010 & 2014):
 - If $X \in \{T, K4, S4, IPC, IPC_{\rightarrow}\}$, then $X^{\textit{CF}} \subseteq X^{\textit{NR}}.$

< □ > < 同 > < 回 > < 回

A Wieldy Tool Failures of MP Between NR fragments of ML's

Properties of CF fragments

• Let $X \in \{T, K4, S4\}$:

- $\phi \in X^{CF}$ iff $\Box \phi \in X^{CF}$ (necessitation).
- $\phi \in X^{CF}$ implies $\phi[p/\psi] \in X^{CF}$ (uniform substitution).
- X^{CF} contains:

$$-\perp \rightarrow p$$

$$-p
ightarrow (q
ightarrow p).$$

$$-(p \rightarrow (q \rightarrow r)) \rightarrow ((p \rightarrow q) \rightarrow (p \rightarrow r))$$

$$-((p \rightarrow q) \rightarrow p) \rightarrow p.$$

$$-\Box(\rho \rightarrow q) \rightarrow (\Box p \rightarrow \Box q).$$

$$-\Box p \rightarrow p$$
 (for T, S4).

$$-\Box p \rightarrow \Box \Box p$$
 (for K4, S4).

- X^{CF} contains all axiom instances in X.
 - Applying uniform substitution to the above.

• $\alpha \rightarrow (\beta \rightarrow \alpha), (\alpha \rightarrow (\beta \rightarrow \gamma)) \rightarrow ((\alpha \rightarrow \beta) \rightarrow (\alpha \rightarrow \gamma)) \in \mathsf{IPC}_{\rightarrow}^{CF}$.

ヘロト 人間 ト ヘヨト ヘヨト

A Wieldy Tool Failures of MP Between NR fragments of ML's

Properties of CF fragments

• Let $X \in \{T, K4, S4\}$:

- $\phi \in X^{CF}$ iff $\Box \phi \in X^{CF}$ (necessitation).
- $\phi \in \mathsf{X}^{CF}$ implies $\phi[\mathbf{p}/\psi] \in \mathsf{X}^{CF}$ (uniform substitution).
- X^{CF} contains:

$$- \bot
ightarrow p.$$

$$-p \rightarrow (q \rightarrow p)$$

$$-(p \rightarrow (q \rightarrow r)) \rightarrow ((p \rightarrow q) \rightarrow (p \rightarrow r))$$

$$-((p \rightarrow q) \rightarrow p) \rightarrow p.$$

$$-\Box(p \rightarrow q) \rightarrow (\Box p \rightarrow \Box q).$$

$$-\Box p \rightarrow p$$
 (for T, S4).

$$-\Box p \rightarrow \Box \Box p$$
 (for K4, S4).

- X^{CF} contains all axiom instances in X.
 - Applying uniform substitution to the above.

• $\alpha \rightarrow (\beta \rightarrow \alpha), (\alpha \rightarrow (\beta \rightarrow \gamma)) \rightarrow ((\alpha \rightarrow \beta) \rightarrow (\alpha \rightarrow \gamma)) \in \mathsf{IPC}_{\rightarrow}^{CF}$.

<ロ> (四) (四) (三) (三) (三)

A Wieldy Tool Failures of MP Between NR fragments of ML's

Properties of CF fragments

- Let $X \in \{T, K4, S4\}$:
 - $\phi \in X^{CF}$ iff $\Box \phi \in X^{CF}$ (necessitation).
 - $\phi \in X^{CF}$ implies $\phi[p/\psi] \in X^{CF}$ (uniform substitution).
 - X^{CF} contains:

$$- \perp \rightarrow p.$$

$$- p \rightarrow (q \rightarrow p).$$

$$- (p \rightarrow (q \rightarrow r)) \rightarrow ((p \rightarrow q) \rightarrow (p \rightarrow r)).$$

$$- ((p \rightarrow q) \rightarrow p) \rightarrow p.$$

$$- \Box (p \rightarrow q) \rightarrow (\Box p \rightarrow \Box q).$$

$$- \Box p \rightarrow p \qquad (for T S4)$$

- $-\Box p \rightarrow p$ (for T, S4).
- $-\Box p \rightarrow \Box \Box p$ (for K4, S4).
- X^{CF} contains all axiom instances in X.
 - Applying uniform substitution to the above.
- $\alpha \rightarrow (\beta \rightarrow \alpha), (\alpha \rightarrow (\beta \rightarrow \gamma)) \rightarrow ((\alpha \rightarrow \beta) \rightarrow (\alpha \rightarrow \gamma)) \in \mathsf{IPC}_{\rightarrow}^{CF}$.

ヘロト ヘアト ヘビト ヘビト

A Wieldy Tool Failures of MP Between NR fragments of ML's

Properties of CF fragments

- Let $X \in \{T, K4, S4\}$:
 - $\phi \in X^{CF}$ iff $\Box \phi \in X^{CF}$ (necessitation).
 - $\phi \in X^{CF}$ implies $\phi[p/\psi] \in X^{CF}$ (uniform substitution).
 - X^{CF} contains:

$$\begin{array}{l} - \bot \rightarrow p. \\ - p \rightarrow (q \rightarrow p). \\ - (p \rightarrow (q \rightarrow r)) \rightarrow ((p \rightarrow q) \rightarrow (p \rightarrow r)) \\ - ((p \rightarrow q) \rightarrow p) \rightarrow p. \\ - \Box (p \rightarrow q) \rightarrow (\Box p \rightarrow \Box q). \end{array}$$

- $-\Box p \rightarrow p$ (for T, S4).
- $-\Box p \rightarrow \Box \Box p$ (for K4, S4).
- X^{CF} contains all axiom instances in X.
 - Applying uniform substitution to the above.

• $\alpha \rightarrow (\beta \rightarrow \alpha), (\alpha \rightarrow (\beta \rightarrow \gamma)) \rightarrow ((\alpha \rightarrow \beta) \rightarrow (\alpha \rightarrow \gamma)) \in \mathsf{IPC}_{\rightarrow}^{CF}$

ヘロン 人間 とくほ とくほ とう

э

A Wieldy Tool Failures of MP Between NR fragments of ML's

Properties of CF fragments

- Let $X \in \{T, K4, S4\}$:
 - $\phi \in X^{CF}$ iff $\Box \phi \in X^{CF}$ (necessitation).
 - $\phi \in X^{CF}$ implies $\phi[p/\psi] \in X^{CF}$ (uniform substitution).
 - X^{CF} contains:

$$\begin{array}{l} - \bot \rightarrow p. \\ - p \rightarrow (q \rightarrow p). \\ - (p \rightarrow (q \rightarrow r)) \rightarrow ((p \rightarrow q) \rightarrow (p \rightarrow r)). \\ - ((p \rightarrow q) \rightarrow p) \rightarrow p. \\ - \Box (p \rightarrow q) \rightarrow (\Box p \rightarrow \Box q). \end{array}$$

- $-\Box p \rightarrow p$ (for T, S4).
- $-\Box p \rightarrow \Box \Box p$ (for K4, S4).
- X^{CF} contains all axiom instances in X.
 - Applying uniform substitution to the above.

•
$$\alpha \rightarrow (\beta \rightarrow \alpha), (\alpha \rightarrow (\beta \rightarrow \gamma)) \rightarrow ((\alpha \rightarrow \beta) \rightarrow (\alpha \rightarrow \gamma)) \in \mathsf{IPC}_{\rightarrow}^{CF}$$

A Wieldy Tool Failures of MP Between NR fragments of ML's

Applied to NR fragments

• Let $X \in \{T, K4, S4\}$:

• X^{NR} contains all axiom instances in X.

– by the fact that $X^{CF} \subseteq X^{NR}$.

• X^{NR} is closed under necessitation.

- directly by Artemov's proof of internalization theorem.

• X^{NR} is not closed under MP.

- otherwise $X^{NR} = X$, contradiction.

• Thus, non-self-referentiality can be abnormal. (Yu 2017)

• IPC^{NR} is not closed under MP.

- $\alpha \rightarrow (\beta \rightarrow \alpha), (\alpha \rightarrow (\beta \rightarrow \gamma)) \rightarrow ((\alpha \rightarrow \beta) \rightarrow (\alpha \rightarrow \gamma)) \in \mathsf{IPC}_{\rightarrow}^{NR}.$ - by the fact that $\mathsf{IPC}_{\rightarrow}^{CF} \subseteq \mathsf{IPC}_{\rightarrow}^{NR}.$
- IPC^{NR} is not closed under *MP*.
 - otherwise IPC $_{\rightarrow}^{NR}$ = IPC $_{\rightarrow}$, contradiction.
- So is IPC^{NR}

A Wieldy Tool Failures of MP Between NR fragments of ML's

Applied to NR fragments

- Let $X \in \{T, K4, S4\}$:
 - X^{NR} contains all axiom instances in X.
 - by the fact that $X^{CF} \subseteq X^{NR}$.
 - X^{NR} is closed under necessitation.
 - directly by Artemov's proof of internalization theorem.
 - X^{NR} is not closed under MP.
 - otherwise $X^{NR} = X$, contradiction.
- Thus, non-self-referentiality can be abnormal. (Yu 2017)
- IPC^{NR} is not closed under MP.
 - $\alpha \rightarrow (\beta \rightarrow \alpha), (\alpha \rightarrow (\beta \rightarrow \gamma)) \rightarrow ((\alpha \rightarrow \beta) \rightarrow (\alpha \rightarrow \gamma)) \in \mathsf{IPC}^{NR}_{\rightarrow}.$ - by the fact that $\mathsf{IPC}^{CF}_{\rightarrow} \subseteq \mathsf{IPC}^{NR}_{\rightarrow}.$
 - IPC^{NR} is not closed under *MP*.
 - otherwise IPC $_{\rightarrow}^{NR}$ = IPC $_{\rightarrow}$, contradiction.
 - So is IPC^{NR}

ヘロト ヘアト ヘビト ヘビト

A Wieldy Tool Failures of MP Between NR fragments of ML's

Applied to NR fragments

- Let $X \in \{T, K4, S4\}$:
 - X^{NR} contains all axiom instances in X.
 - by the fact that $X^{CF} \subseteq X^{NR}$.
 - X^{NR} is closed under necessitation.
 - directly by Artemov's proof of internalization theorem.
 - X^{NR} is not closed under MP.
 - otherwise $X^{NR} = X$, contradiction.

• Thus, non-self-referentiality can be abnormal. (Yu 2017)

- IPC^{NR} is not closed under MP.
 - $\alpha \rightarrow (\beta \rightarrow \alpha), (\alpha \rightarrow (\beta \rightarrow \gamma)) \rightarrow ((\alpha \rightarrow \beta) \rightarrow (\alpha \rightarrow \gamma)) \in \mathsf{IPC}^{NR}_{\rightarrow}.$ - by the fact that $\mathsf{IPC}^{CF}_{\rightarrow} \subseteq \mathsf{IPC}^{NR}_{\rightarrow}.$
 - IPC^{*NR*} is not closed under *MP*.
 - otherwise IPC $_{\rightarrow}^{NR}$ = IPC $_{\rightarrow}$, contradiction.
 - So is IPC^{NR}

ヘロン 人間 とくほ とくほ とう

-

A Wieldy Tool Failures of MP Between NR fragments of ML's

Applied to NR fragments

- Let $X \in \{T, K4, S4\}$:
 - X^{NR} contains all axiom instances in X.
 - by the fact that $X^{CF} \subseteq X^{NR}$.
 - X^{NR} is closed under necessitation.
 - directly by Artemov's proof of internalization theorem.
 - X^{NR} is not closed under MP.
 - otherwise $X^{NR} = X$, contradiction.
- Thus, non-self-referentiality can be abnormal. (Yu 2017)
- IPC^{NR} is not closed under MP.
 - $\alpha \rightarrow (\beta \rightarrow \alpha), (\alpha \rightarrow (\beta \rightarrow \gamma)) \rightarrow ((\alpha \rightarrow \beta) \rightarrow (\alpha \rightarrow \gamma)) \in \mathsf{IPC}_{\rightarrow}^{NR}$. - by the fact that $\mathsf{IPC}_{\rightarrow}^{CF} \subseteq \mathsf{IPC}_{\rightarrow}^{NR}$.
 - IPC_{\rightarrow}^{NR} is not closed under *MP*.
 - otherwise $IPC_{\rightarrow}^{NR} = IPC_{\rightarrow}$, contradiction.
 - So is IPC^{NR}

A Wieldy Tool Failures of MP Between NR fragments of ML's

Applied to NR fragments

- Let $X \in \{T, K4, S4\}$:
 - X^{NR} contains all axiom instances in X.
 - by the fact that $X^{CF} \subseteq X^{NR}$.
 - X^{NR} is closed under necessitation.
 - directly by Artemov's proof of internalization theorem.
 - X^{NR} is not closed under MP.
 - otherwise $X^{NR} = X$, contradiction.
- Thus, non-self-referentiality can be abnormal. (Yu 2017)
- IPC^{NR} is not closed under MP.
 - $\alpha \rightarrow (\beta \rightarrow \alpha), (\alpha \rightarrow (\beta \rightarrow \gamma)) \rightarrow ((\alpha \rightarrow \beta) \rightarrow (\alpha \rightarrow \gamma)) \in \mathsf{IPC}_{\rightarrow}^{NR}.$ - by the fact that $\mathsf{IPC}_{\rightarrow}^{CF} \subseteq \mathsf{IPC}_{\rightarrow}^{NR}.$
 - IPC_{\rightarrow}^{NR} is not closed under *MP*.
 - otherwise $IPC_{\rightarrow}^{\textit{NR}} = IPC_{\rightarrow}$, contradiction.
 - So is IPC^{NR}.

(日)

A Wieldy Tool Failures of *MP* Between *NR* fragments of ML's

Between NR fragments of ML's

• Easy to show are:

- $T^{NR} \subseteq S4^{NR}$ and
- $K4^{NR} \subseteq S4^{NR}$
 - though we will not give a proof here...
- Hard to show is:
 - there are no more inclusions!
- Therefore, when going from a smaller ML to a greater ML, non-self-referentiality is not always conservative. (Yu 2017)

A Wieldy Tool Failures of *MP* Between *NR* fragments of ML's

Between NR fragments of ML's

• Easy to show are:

- $T^{NR} \subseteq S4^{NR}$ and
- $K4^{NR} \subseteq S4^{NR}$
 - though we will not give a proof here...
- Hard to show is:
 - there are no more inclusions!
- Therefore, when going from a smaller ML to a greater ML, non-self-referentiality is not always conservative. (Yu 2017)

A Wieldy Tool Failures of *MP* Between *NR* fragments of ML's

Between NR fragments of ML's

- Easy to show are:
 - $T^{NR} \subseteq S4^{NR}$ and
 - K4^{*NR*} ⊆ S4^{*NR*}
 - though we will not give a proof here...
- Hard to show is:
 - there are no more inclusions!
- Therefore, when going from a smaller ML to a greater ML, non-self-referentiality is not always conservative. (Yu 2017)

A Wieldy Tool Failures of *MP* Between *NR* fragments of ML's

Between NR fragments of ML's (continued)



俞珺华 (Yu, Junhua) Self-referentiality in the framework of justification logics

イロト イポト イヨト イヨト

ъ

A Wieldy Tool Failures of *MP* Between *NR* fragments of ML's

Between NR fragments of ML's (contiinued)



俞珺华 (Yu, Junhua) Self-referentiality in the framework of justification logics

イロト イポト イヨト イヨト

ъ

A Wieldy Tool Failures of *MP* Between *NR* fragments of ML's

Between NR fragments of ML's (contiiinued)



イロト イポト イヨト イヨト

3

Realization in JL A Wi Self-referentiality Failu Properties of *NR* Fragments Betw

A Wieldy Tool Failures of *MP* Between *NR* fragments of ML's

Between NR fragments of ML's (contivnued)



A Wieldy Tool Failures of *MP* Between *NR* fragments of ML's

Between NR fragments of ML's (contvnued)



Realization in JL A Wieldy Self-referentiality Failures Properties of *NR* Fragments Between

A Wieldy Tool Failures of *MP* Between *NR* fragments of ML's

Between NR fragments of ML's (contvinued)



Realization in JL A Wieldy Tool Self-referentiality Failures of *MP* Properties of *NR* Fragments Between *NR* fragments of ML's

Between NR fragments of ML's (contviinued)



Realization in JL A Wieldy Tool Self-referentiality Failures of MP Properties of NR Fragments Between NR fragments of ML's

Between NR fragments of ML's (contviiinued)



Realization in JL A Wield Self-referentiality Failures Properties of NR Fragments Betwee

A Wieldy Tool Failures of *MP* Between *NR* fragments of ML's

Between NR fragments of ML's (contixnued)



Realization in JL A Wie Self-referentiality Failure Properties of *NR* Fragments Betwee

A Wieldy Tool Failures of *MP* Between *NR* fragments of ML's

Between NR fragments of ML's (contxnued)



P.S.: Not all instances come from Kuznets' κ 's, e.g., let $\iota = \Diamond \Box p \rightarrow \Diamond \Box \Diamond p$.

Realization in JL	A Wieldy Tool
Self-referentiality	Failures of MP
Properties of NR Fragments	Between NR fragments of ML's

Thanks!

俞珺华 (Yu, Junhua) Self-referentiality in the framework of justification logics

◆□→ ◆□→ ◆三→ ◆三→

き のへで