# Group Announcement Logic

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# Group Announcement Logic: background

- Two current trends in logics for multi-agent interaction:

  1. Logics of coalitional ability (Coalition Logic, ATL, Stit logic, ...)

     - Recent interest: incomplete information

  2. Dynamic epistemic logic

     - Epistemic pre- and post- conditions of actions

     - Recent interest: quantification over formulae (APAL, ...)

- We combine ideas from both in order to analyse the logic of *group announcements*

# Elevator pitch

Group Announcement Logic extends public announcement logic with:

$$\langle G\rangle\phi : \text{"Group } G \text{ can make an announcement after which } \phi \text{ is true"}$$

# Public Announcement Logic (Plaza, 1989)

$$\varphi ::= p \mid K_i\varphi \mid \neg\varphi \mid \varphi_1 \wedge \varphi_2 \mid \langle\varphi_1\rangle\varphi_2$$

$\phi_1$ is true, and $\phi_2$ is true after $\phi_1$ is announced

Formally:

$M = (S, \sim_1, \dots, \sim_n, V)$ $\quad \sim_i$ equivalence rel. over S

$$M, s \models K_i\phi \quad \Leftrightarrow \quad \forall t \sim_i s \; M, t \models \phi$$

$$M, s \models \langle\phi_1\rangle\phi_2 \quad \Leftrightarrow \quad M, s \models \phi_1 \text{ and } M|\phi_1, s \models \phi_2$$

The model resulting from removing states where $\phi_1$ is false

# Adding quantification: APAL

$$M, s \models \langle \phi_1 \rangle \phi_2 \Leftrightarrow M, s \models \phi_1 \text{ and } M|\phi_1, s \models \phi_2$$

Idea (van Benthem, Analysis, 2004): interpret the modal diamond as "there is an announcement such that.."

Arbitrary announcement logic (APAL) (Balbiani et al., TARK 2007):

$$\varphi ::= p \mid K_i\varphi \mid \neg\varphi \mid \varphi_1 \wedge \varphi_2 \mid \langle \varphi_1 \rangle \varphi_2 \mid \Diamond\phi$$

$$M, s \models \Diamond\phi \Leftrightarrow \exists\psi \; M, s \models \langle \psi \rangle \phi$$

# Quantification in APAL

$$M, s \models \Diamond\phi \Leftrightarrow \exists\psi\ M, s \models \langle\psi\rangle\phi$$

Note: the quantification includes announcements that cannot be truthfully made in the system

# Quantification: announcements by an agent

$$K_i \psi$$

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$$M, s \models \langle i \rangle \phi \Leftrightarrow \exists \psi \;\; M, s \models \langle K_i \psi \rangle \phi$$

# Quantification: announcements by a group

$$M, s \models \langle G \rangle \phi \quad \Leftrightarrow \quad \exists \{\psi_i : i \in G\}\ M, s \models \langle \textstyle\bigwedge_{i \in G} K_i \psi \rangle \phi$$

Group Announcement Logic (GAL):

$$\varphi ::= p \mid K_i \varphi \mid \neg \varphi \mid \varphi_1 \wedge \varphi_2 \mid \langle \varphi_1 \rangle \varphi_2 \mid \langle G \rangle \phi$$

# The Russian Cards Puzzle

*From a pack of seven known cards* $0, 1, 2, 3, 4, 5, 6$ *Alice (a) and Bob (b) each draw three cards and Eve (c) gets the remaining card. How can Alice and Bob openly (publicly) inform each other about their cards, without Eve learning of any of their cards who holds it?*

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Suppose Alice draws $\{0, 1, 2\}$, Bob draws $\{3, 4, 5\}$, and Eve 6.

# Formalisation

**Model**

Players only know their own cards.

**Logic**

| | | |
|---|---|---|
| $q_a$ | | agent $a$ holds card $q$. |
| $ijk_a$ | $(i_a \wedge j_a \wedge k_a)$ | agent $a$'s hand of cards is $\{i, j, k\}$. |

**Epistemic postconditions**

| | | |
|---|---|---|
| Bob informs Alice | $\mathsf{aknowsbs}$ | $\bigwedge(ijk_b \rightarrow K_a ijk_b)$ |
| Alice informs Bob | $\mathsf{bknowsas}$ | $\bigwedge(ijk_a \rightarrow K_b ijk_a)$ |
| Eve remains ignorant | $\mathsf{cignorant}$ | $\bigwedge(\neg K_c q_a \wedge \neg K_c q_b)$ |

# Solution?

$Alice\ says\ \text{``}I\ have\ \{0,1,2\}\ or\ Bob\ has\ \{0,1,2\}.\text{''}$

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~~*Alice says "I have* $\{0,1,2\}$ *or Bob has* $\{0,1,2\}$*."*~~

*The teacher says "Alice has* $\{0,1,2\}$ *or Bob has* $\{0,1,2\}$*."*

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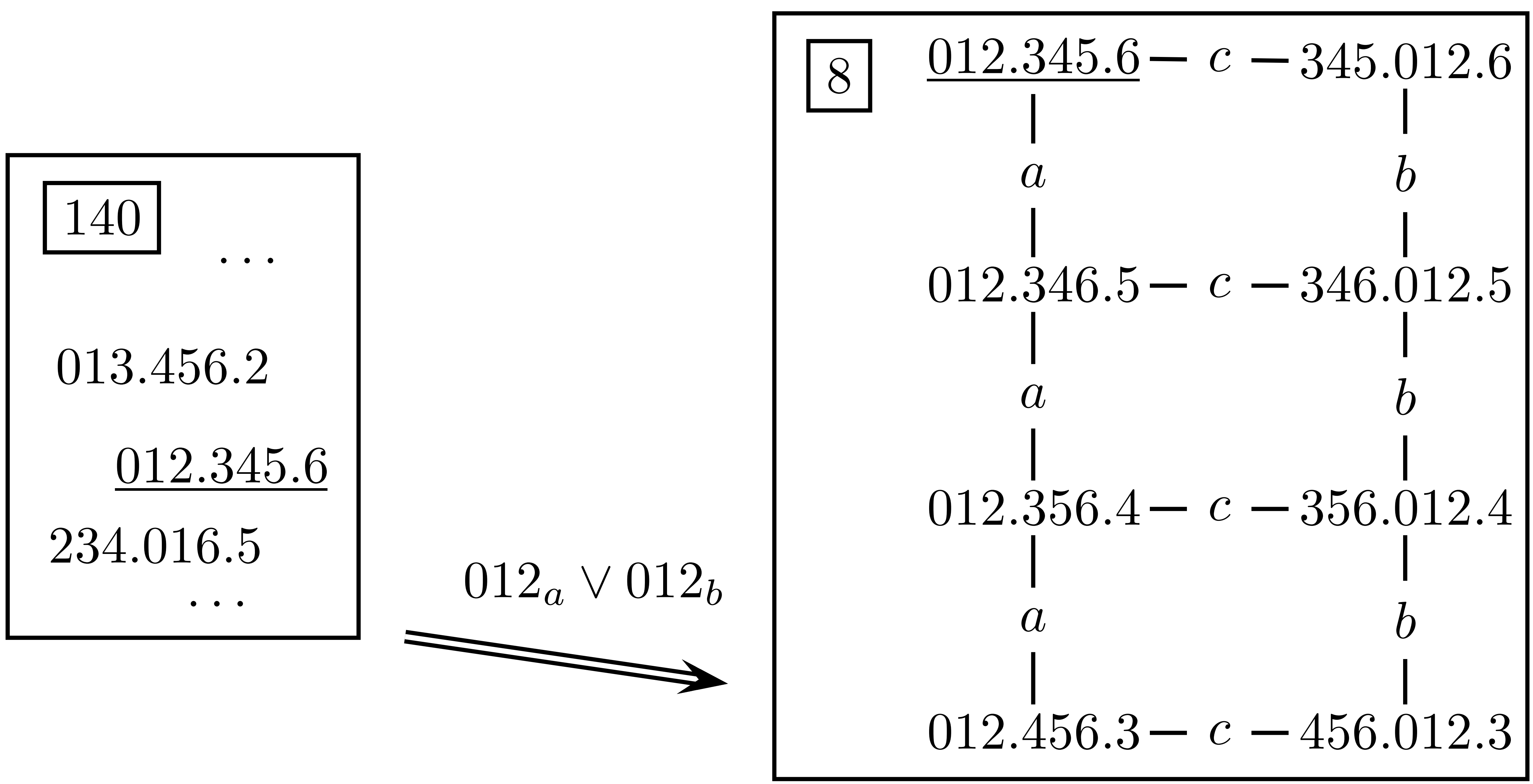

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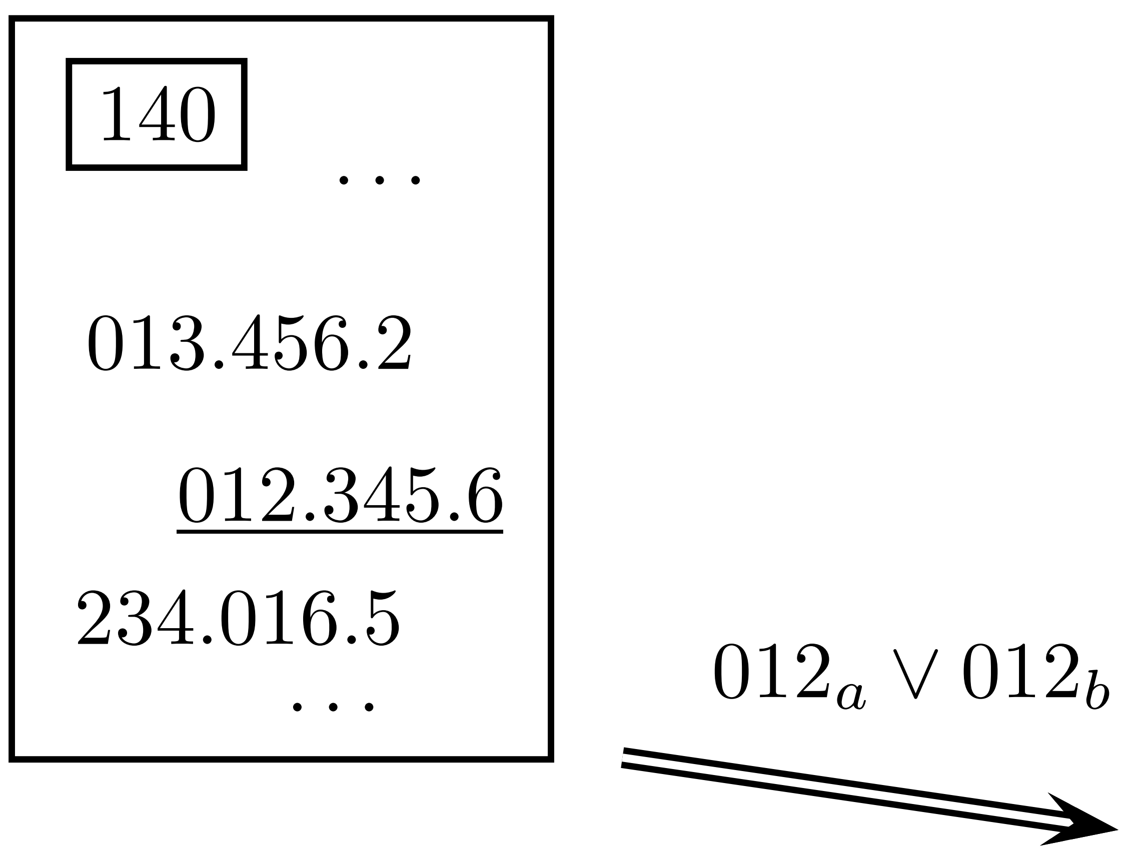
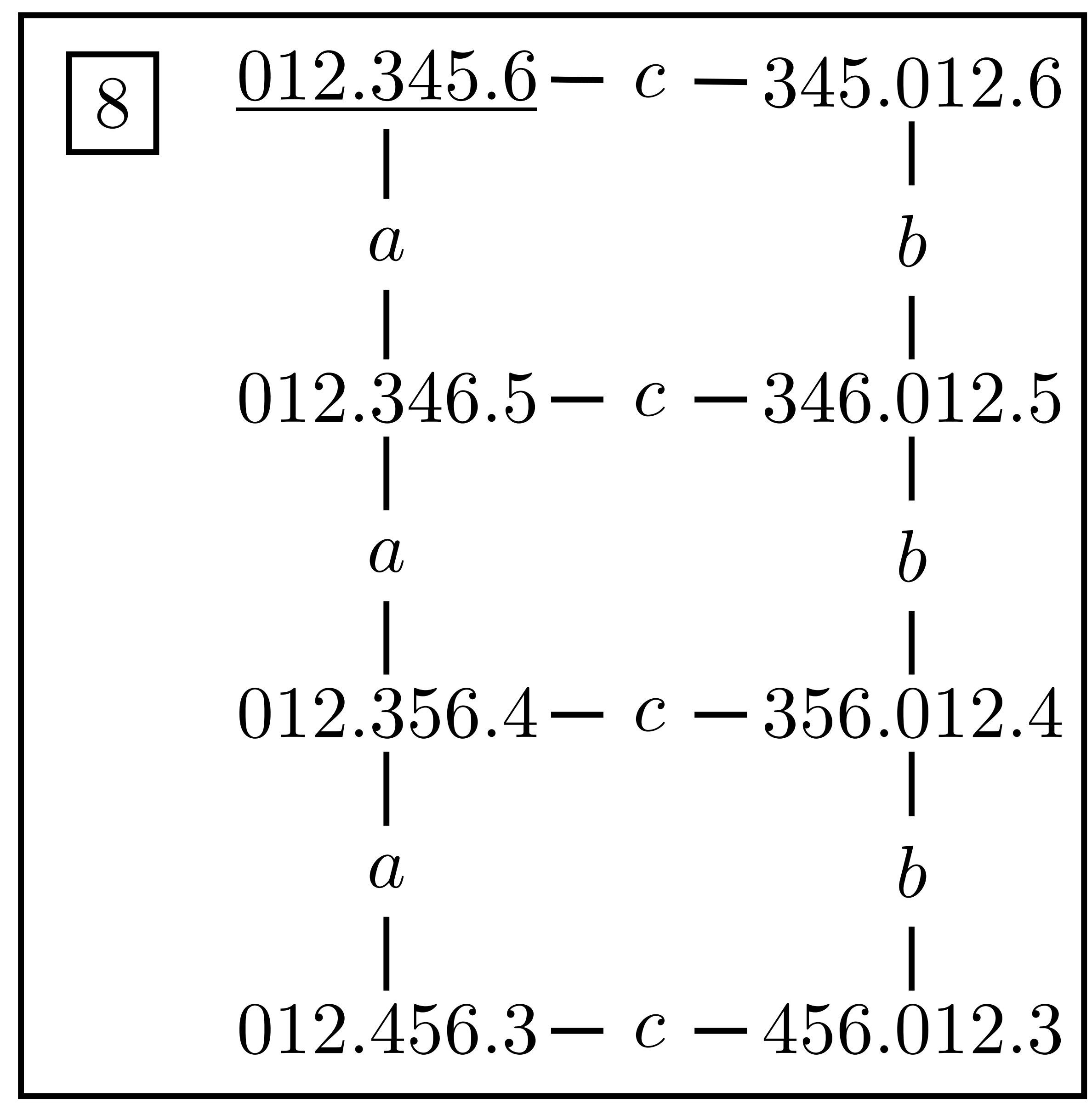
$[012_a \vee 012_b]$cignorant



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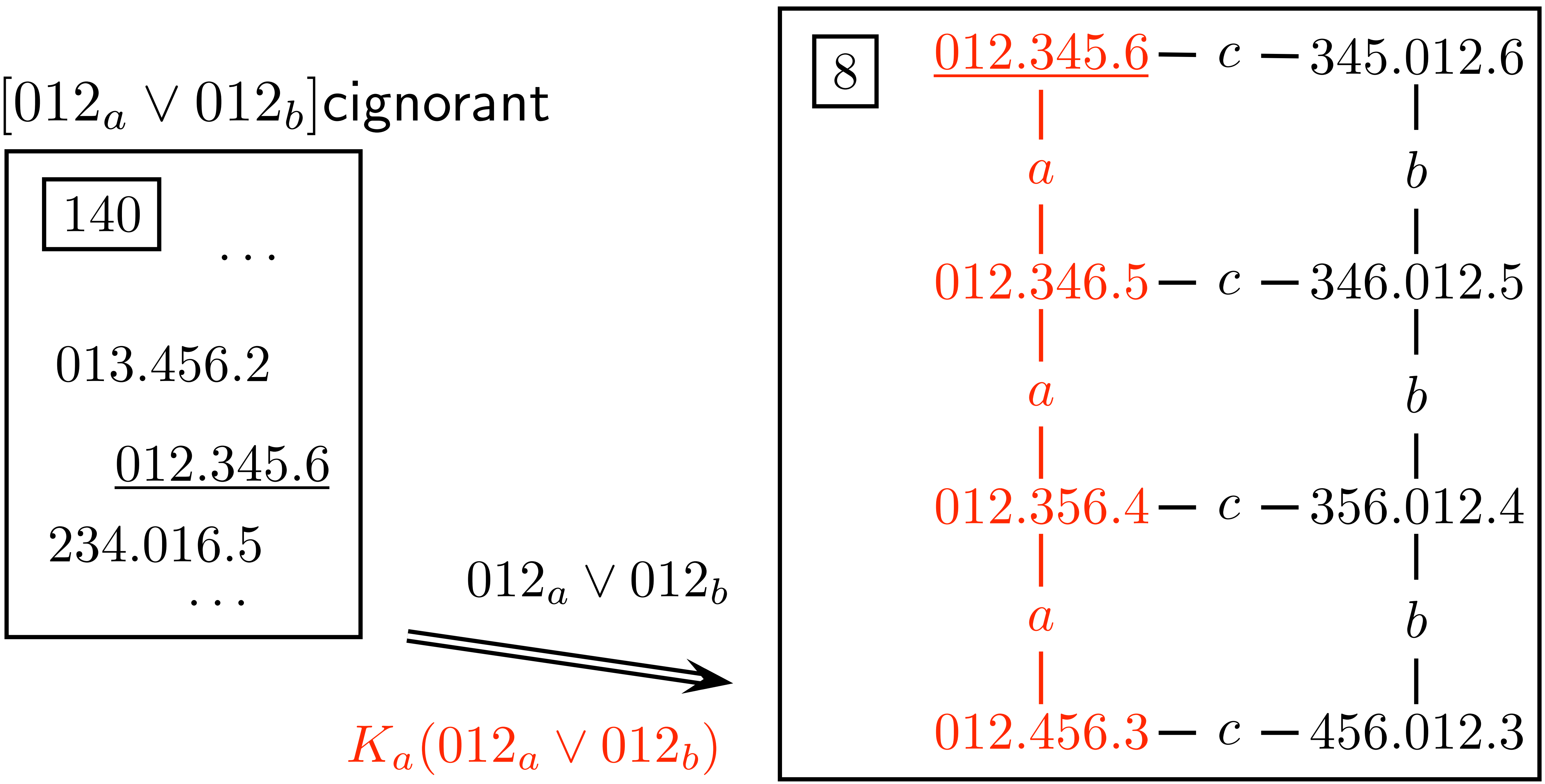

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*The teacher says "Alice has $\{0,1,2\}$ or Bob has $\{0,1,2\}$."*

$\neg[K_a(012_a \vee 012_b)]$cignorant

$[012_a \vee 012_b]$cignorant

$\boxed{140}$

$\dots$

$013.456.2$

$\underline{012.345.6}$

$234.016.5$

$\dots$

$$\xRightarrow[K_a(012_a \vee 012_b)]{012_a \vee 012_b}$$

$\boxed{8}$

$$\begin{tikzcd}
\underline{012.345.6} \arrow[r, dash, "c"] \arrow[d, dash, "a"] & 345.012.6 \arrow[d, dash, "b"] \\
012.346.5 \arrow[r, dash, "c"] \arrow[d, dash, "a"] & 346.012.5 \arrow[d, dash, "b"] \\
012.356.4 \arrow[r, dash, "c"] \arrow[d, dash, "a"] & 356.012.4 \arrow[d, dash, "b"] \\
012.456.3 \arrow[r, dash, "c"] & 456.012.3
\end{tikzcd}$$

# Try 2

*Alice says "I don't have card 6."*

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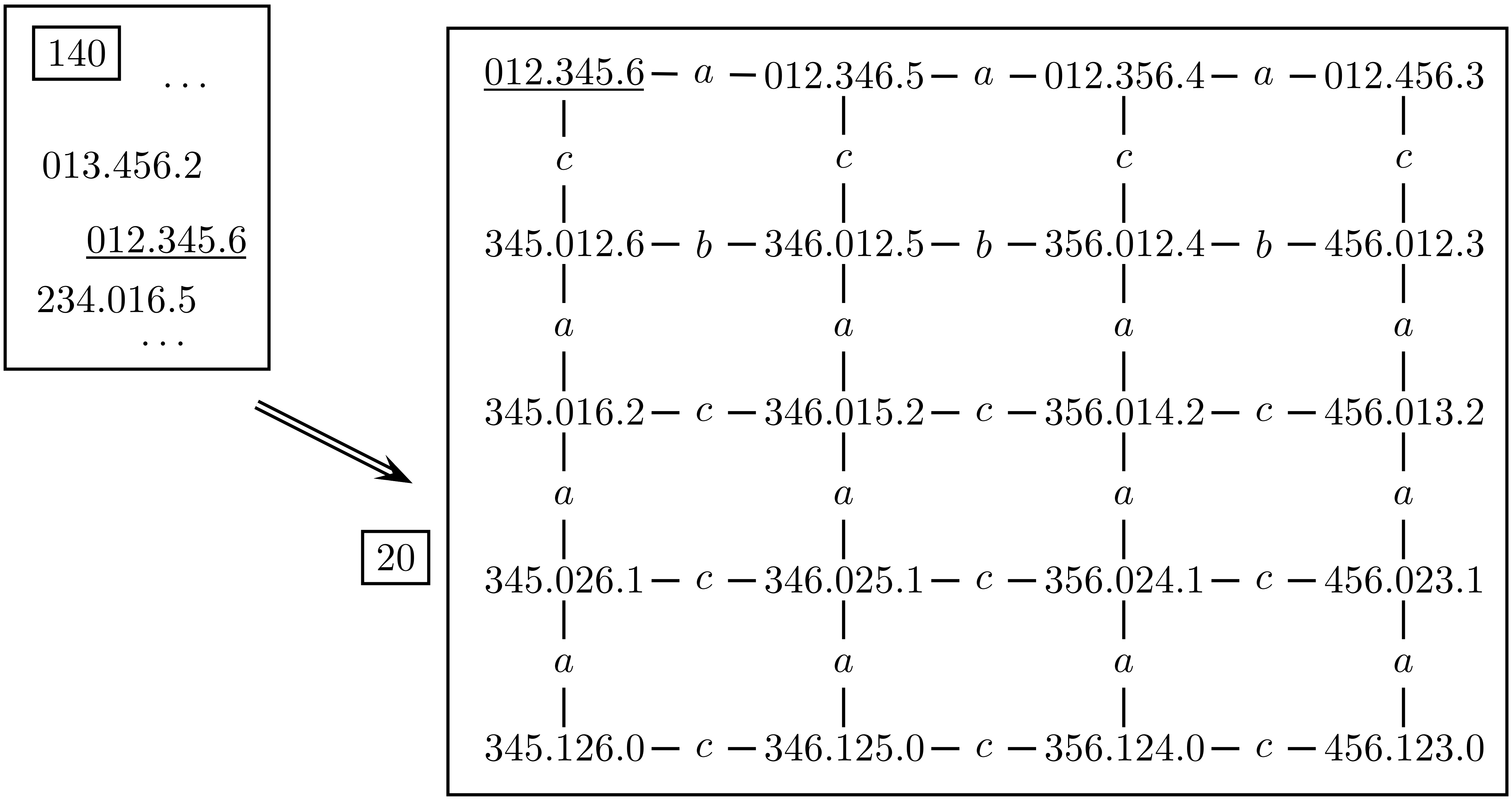

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$$\neg[K_a(012_a \vee \neg(0_a \vee 1_a \vee 2_a))]K_cK_a\text{cignorant}$$

$\boxed{140}$ …

013.456.2

<u>012.345.6</u>

234.016.5

…

$$\begin{tikzcd}
\underline{012.345.6} \arrow[r, dash, "a"] \arrow[d, dash, "c"] & 012.346.5 \arrow[r, dash, "a"] \arrow[d, dash, "c"] & 012.356.4 \arrow[r, dash, "a"] \arrow[d, dash, "c"] & 012.456.3 \arrow[d, dash, "c"] \\
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345.016.2 \arrow[r, dash, "c"] & 346.015.2 \arrow[r, dash, "c"] & 356.014.2 \arrow[r, dash, "c"] & 456.013.2
\end{tikzcd}$$

Eve doesn't know that Alice knows that Eve is ignorant. But it is reasonable that Eve assumes that Alice knows that Eve is ignorant – she knows that Ann wants to keep her secret. But that is informative for Eve!

$$\begin{tikzcd}
345.126.0 \arrow[r, dash, "c"] & 346.125.0 \arrow[r, dash, "c"] & 356.124.0 \arrow[r, dash, "c"] & 456.123.0
\end{tikzcd}$$

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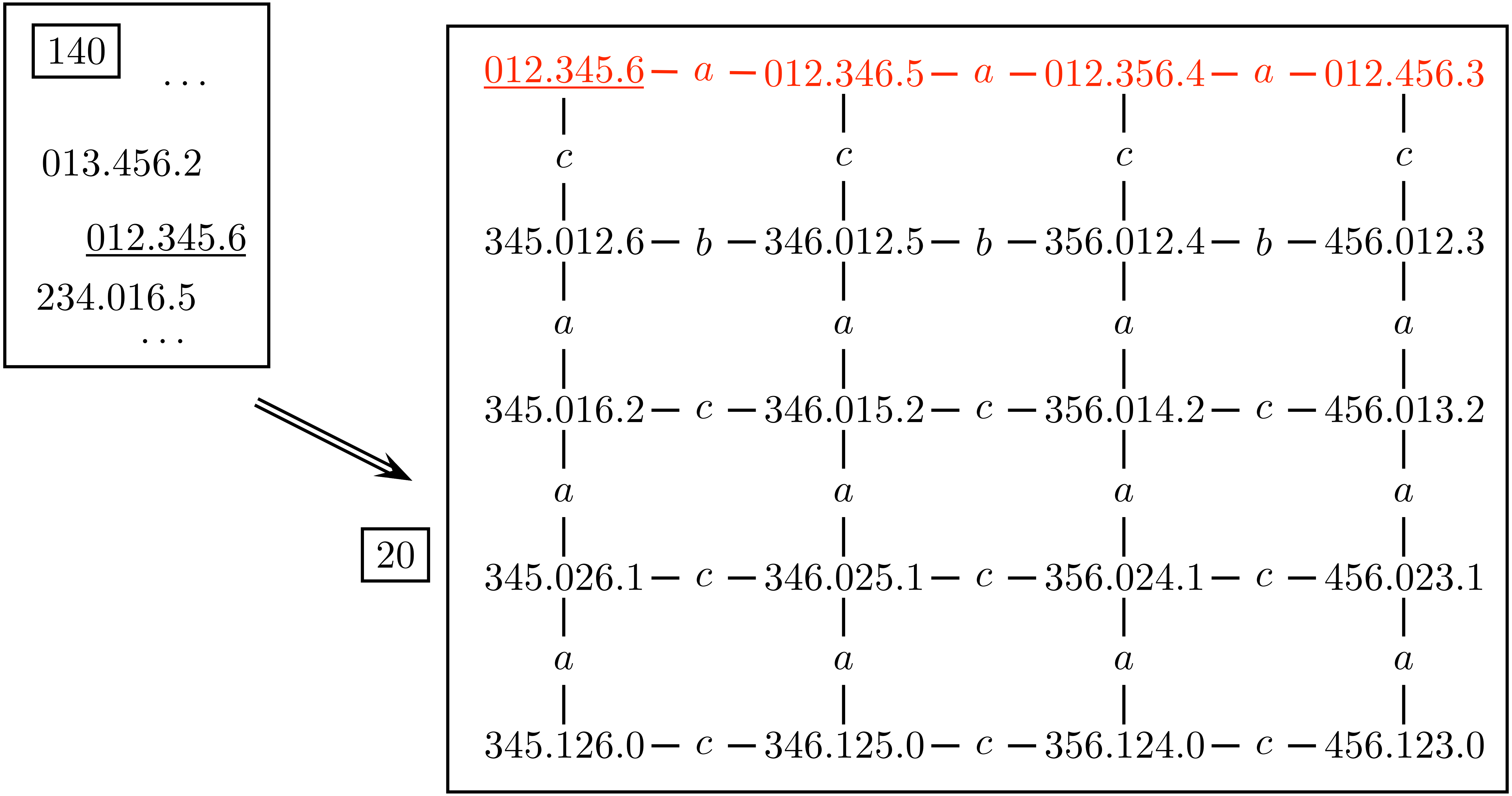


$$[K_a(012_a \vee \neg(0_a \vee 1_a \vee 2_a))][K_a \text{cignorant}]\neg K_a \text{cignorant}$$

# Solution

**Solution**: Alice says "I have one of 012, 034, 056, 135, 246," and Bob then says "Eve has card 6."

H. van Ditmarsch, *The Russian Cards Problem*, Studia Logica **75**, 2003

# Example: **The Russian Cards Problem**

From a pack of seven known cards 0,1,2,3,4,5,6 Anne and Bill each draw three cards and Cath gets the remaining card. How can Anne and Bill openly inform each other about their cards, without Cath learning who holds any of their cards?

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Formalisation: $012_a$ : "Ann has cards 0,1 and 2"

$$(one)\ \bigwedge_{ijk}(ijk_b \rightarrow K_a ijk_b) \quad (two)\ \bigwedge_{ijk}(ijk_a \rightarrow K_b ijk_a)$$

$$(three)\ \bigwedge_{q=0}^{6}((q_a \rightarrow \neg K_c q_a) \wedge (q_b \rightarrow \neg K_c q_b))$$

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Known solution:

$$anne \equiv 012_a \vee 034_a \vee 056_a \vee 135_a \vee 246_a$$

$$bill \equiv 345_b \vee 125_b \vee 024_b$$

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PAL: $\langle K_a anne \rangle \langle K_b bill \rangle (one \wedge two \wedge three)$

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GAL: $\langle a\rangle\langle b\rangle(one \wedge two \wedge three)$

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# Quantification: sequences of announcements

APAL: $\Diamond\Diamond\phi \rightarrow \Diamond\phi$

announcing in sequence $\psi, \chi$ $\Leftrightarrow$ announcing $\psi \wedge [\psi]\chi$

GAL: $\langle G\rangle\langle G\rangle\phi \rightarrow \langle G\rangle\phi$?

# Quantification: sequences of announcements

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GAL: $\langle G\rangle\langle G\rangle\phi \to \langle G\rangle\phi$?

$\bigwedge K_i\psi, \bigwedge K_i\chi \Leftrightarrow \bigwedge K_i\psi \land [\bigwedge K_i\psi] \bigwedge K_i\chi$

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not a group announcement

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not a group announcement

**Theorem**: Yes.

# Quantification: sequences of announcements

$$\langle G\rangle\langle G\rangle\phi \rightarrow \langle G\rangle\phi$$

$$M, s \models \langle G\rangle\phi \Leftrightarrow \text{ there is an announcement by } G\text{, after which } \phi$$

# Quantification: sequences of announcements

$$\langle G\rangle\langle G\rangle\phi \rightarrow \langle G\rangle\phi$$

$M, s \models \langle G\rangle\phi \Leftrightarrow$ there is an announcement by $G$, after which $\phi$

$\Leftrightarrow$ there is a sequence of announcements by $G$, after which $\phi$

# Example: **Russian Cards** (ctnd.)

$$\langle K_a anne\rangle\langle K_b bill\rangle(one \wedge two \wedge three)$$

$$\langle a\rangle\langle b\rangle(one \wedge two \wedge three)$$

$$\langle ab\rangle(one \wedge two \wedge three)$$

# Knowledge and Ability: general actions

- Consider the general case that agents have arbitrary joint actions (and not only group announcements) available, that will take the system to a new state

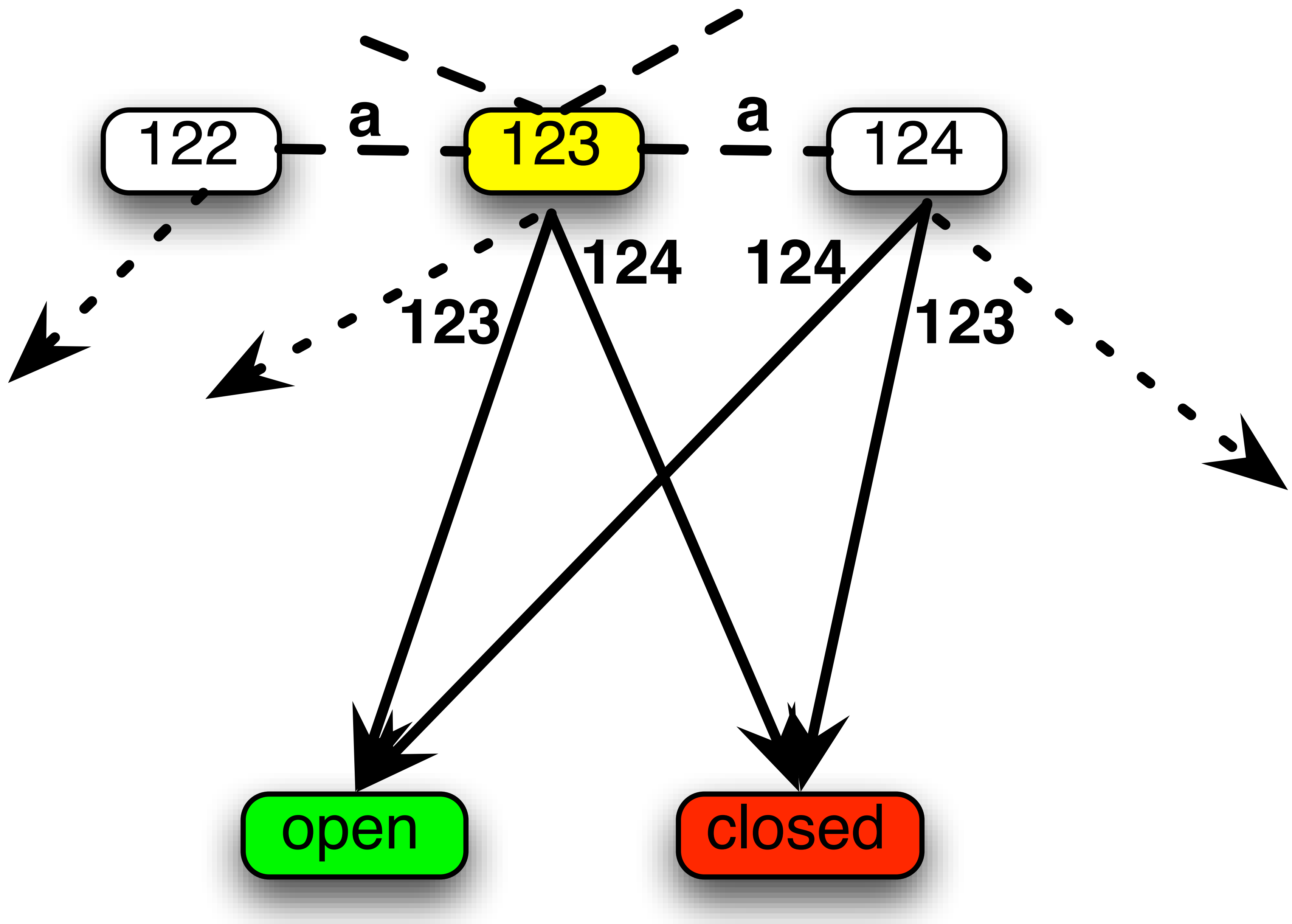
- Two variants of ability under incomplete information:

  - Knowing *de dicto* that you can achive something: in all the states you consider possible, you can achive the goal (by performing some action)

  - Knowing *de re* that you can achieve something: there is some action which will achieve the goal in all the states you consider possible

# Knowledge and Ability: general actions

- Example: agent in front of a combination-lock safe; does not know the combination; correct combination is 123



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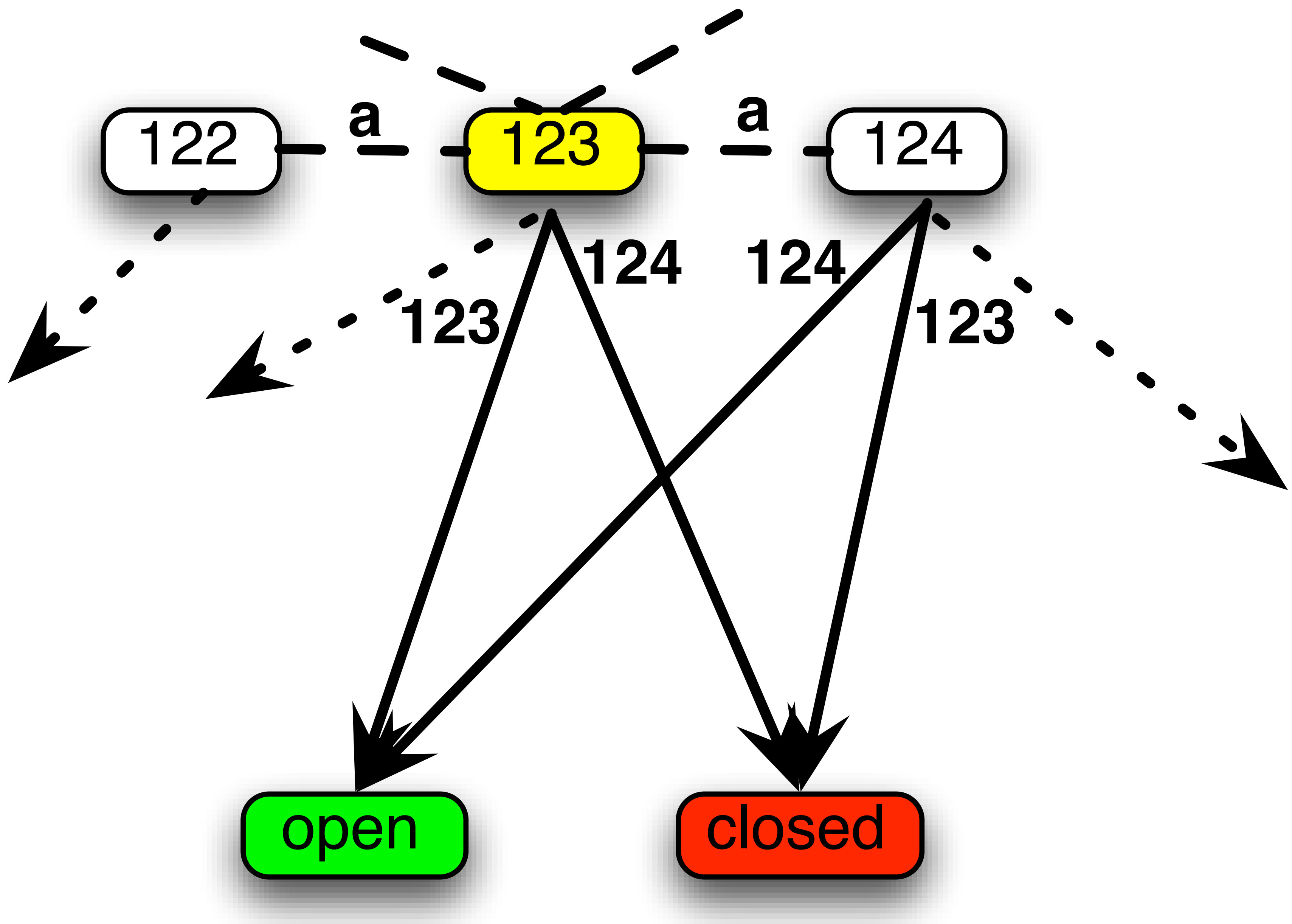


$\langle a \rangle \, open$

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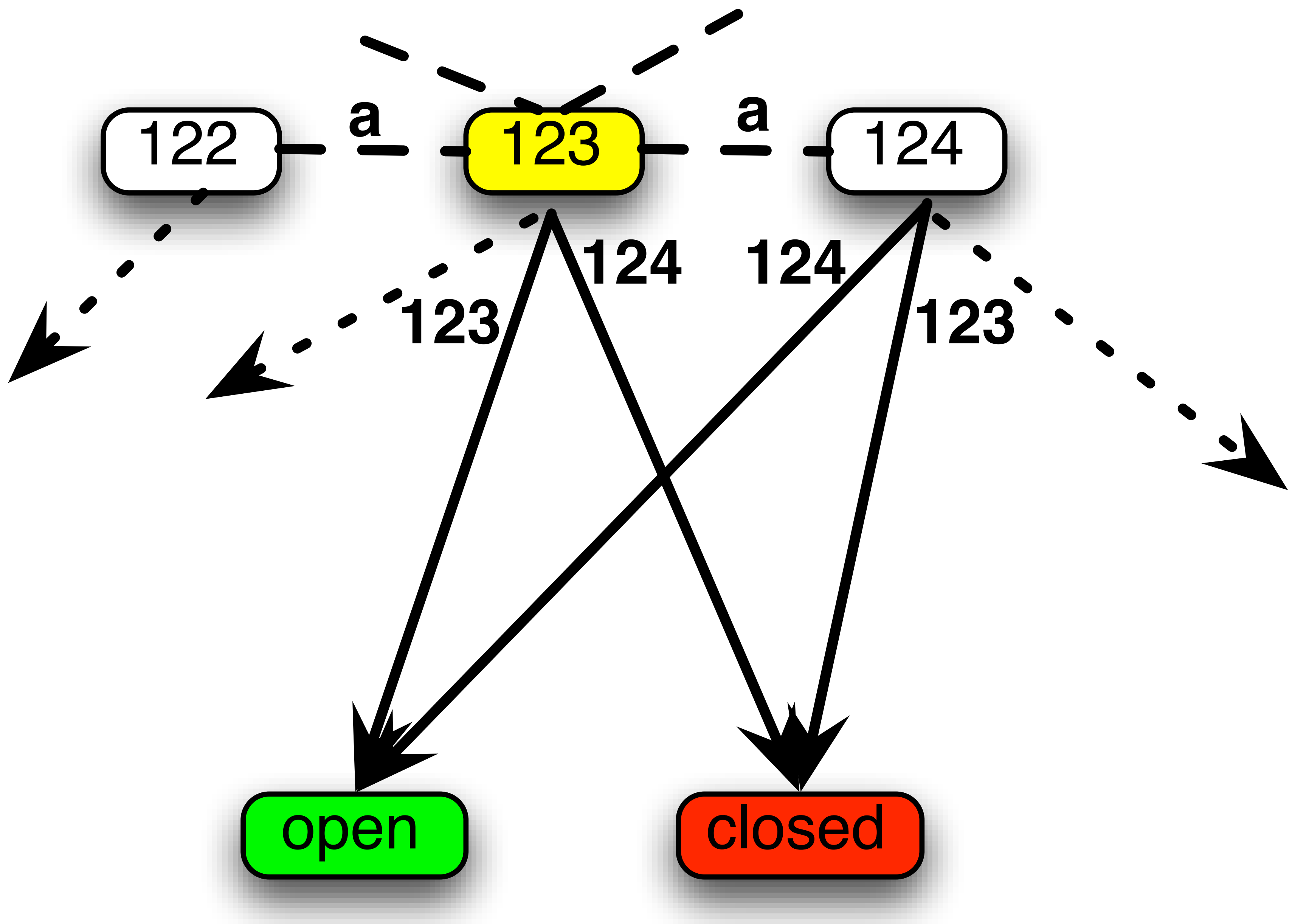


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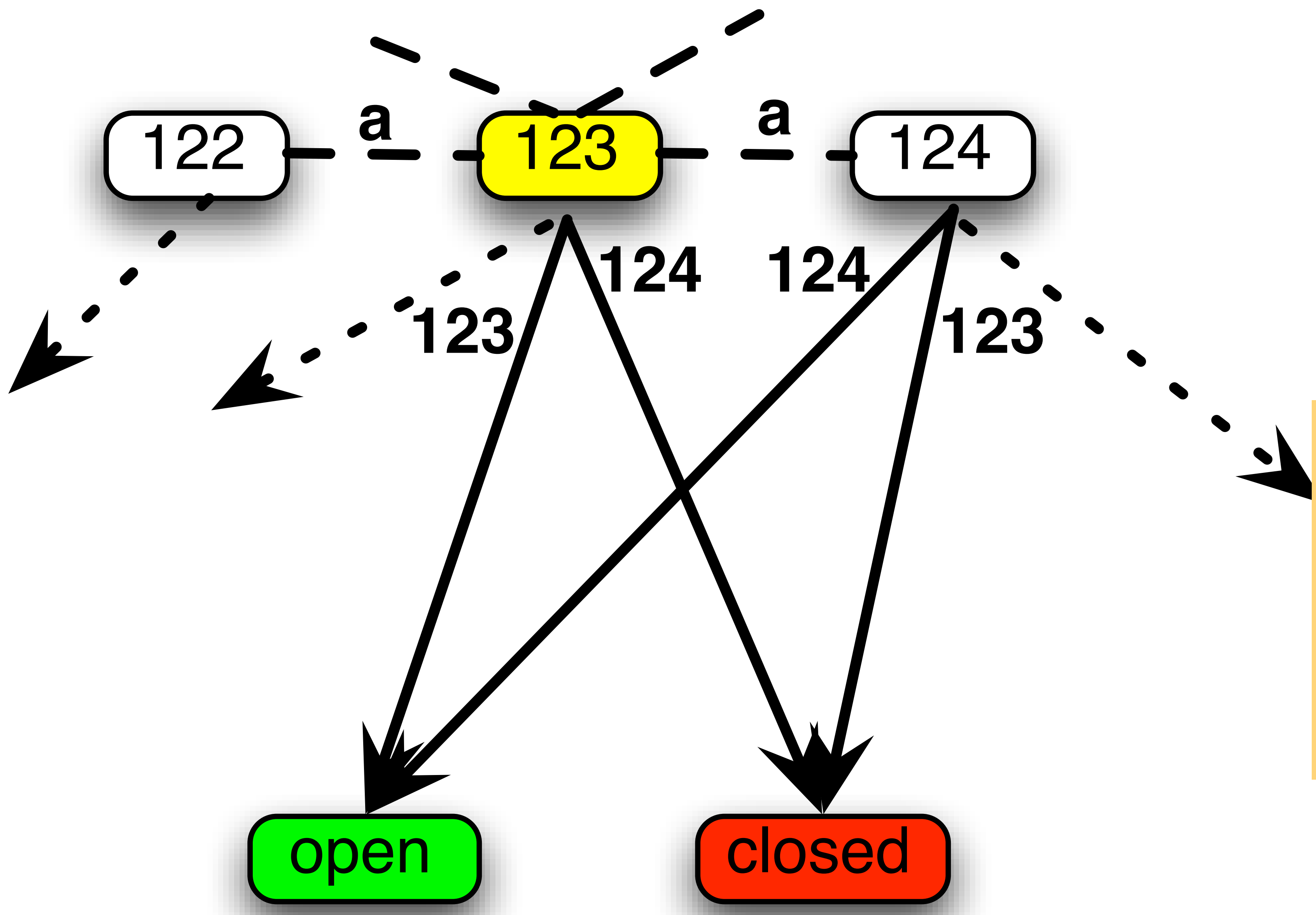



$$\langle a \rangle open$$

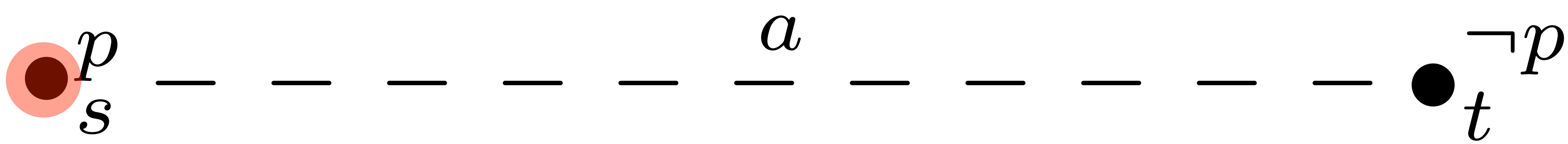
$$K_a \langle a \rangle open$$

**But *a* does not know *de re* that she can open the safe**

# Knowledge and Ability: general actions

- It turns out that knowledge of ability *de re* is not expressible in standard logics combining knowledge and ability
    - Alternating-time Temporal Epistemic Logic (ATEL)  (van der Hoek & Wooldridge)
- Several recent works, e.g. (Jamroga and van der Hoek, 2004), (Jamroga and Ågotnes, 2007), have focused on extending ATEL to be able to express knowledge *de re*
- In GAL, knowledge and action are intimately connected
    - How do the previous observations apply to GAL?

# Being able to without knowing it




$$s \models \langle a \rangle p \wedge \neg K_a \langle a \rangle p$$

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$$K_a \langle ab \rangle (one \wedge two \wedge three)$$

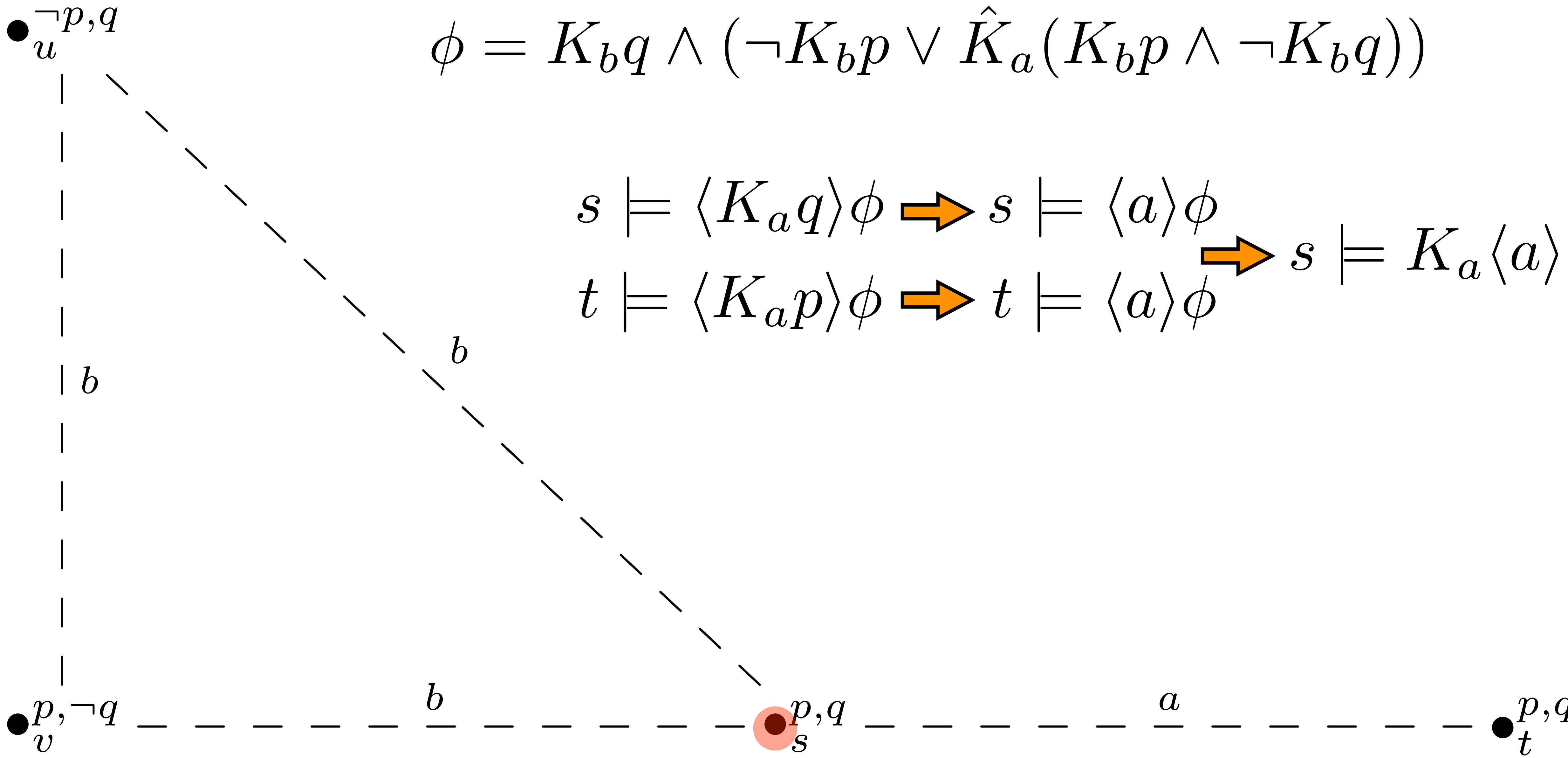
# Being able to, knowing that, but not knowing how

$$\phi = K_b q \wedge (\neg K_b p \vee \hat{K}_a(K_b p \wedge \neg K_b q))$$

$$s \models \langle K_a q \rangle \phi \Rightarrow s \models \langle a \rangle \phi$$

$$t \models \langle K_a p \rangle \phi \Rightarrow t \models \langle a \rangle \phi$$

$$\Rightarrow s \models K_a \langle a \rangle \phi$$



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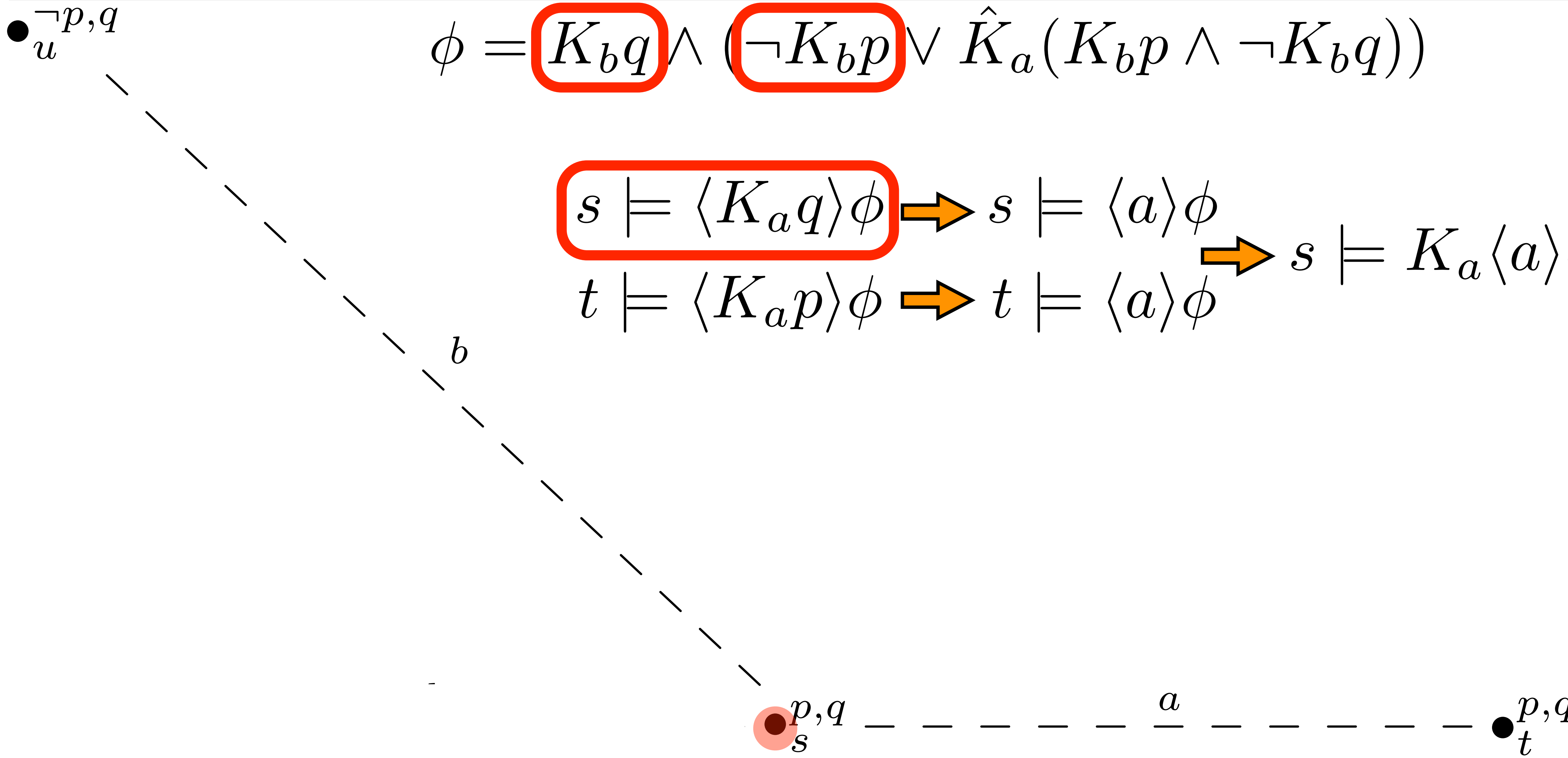


$$s \models \phi$$

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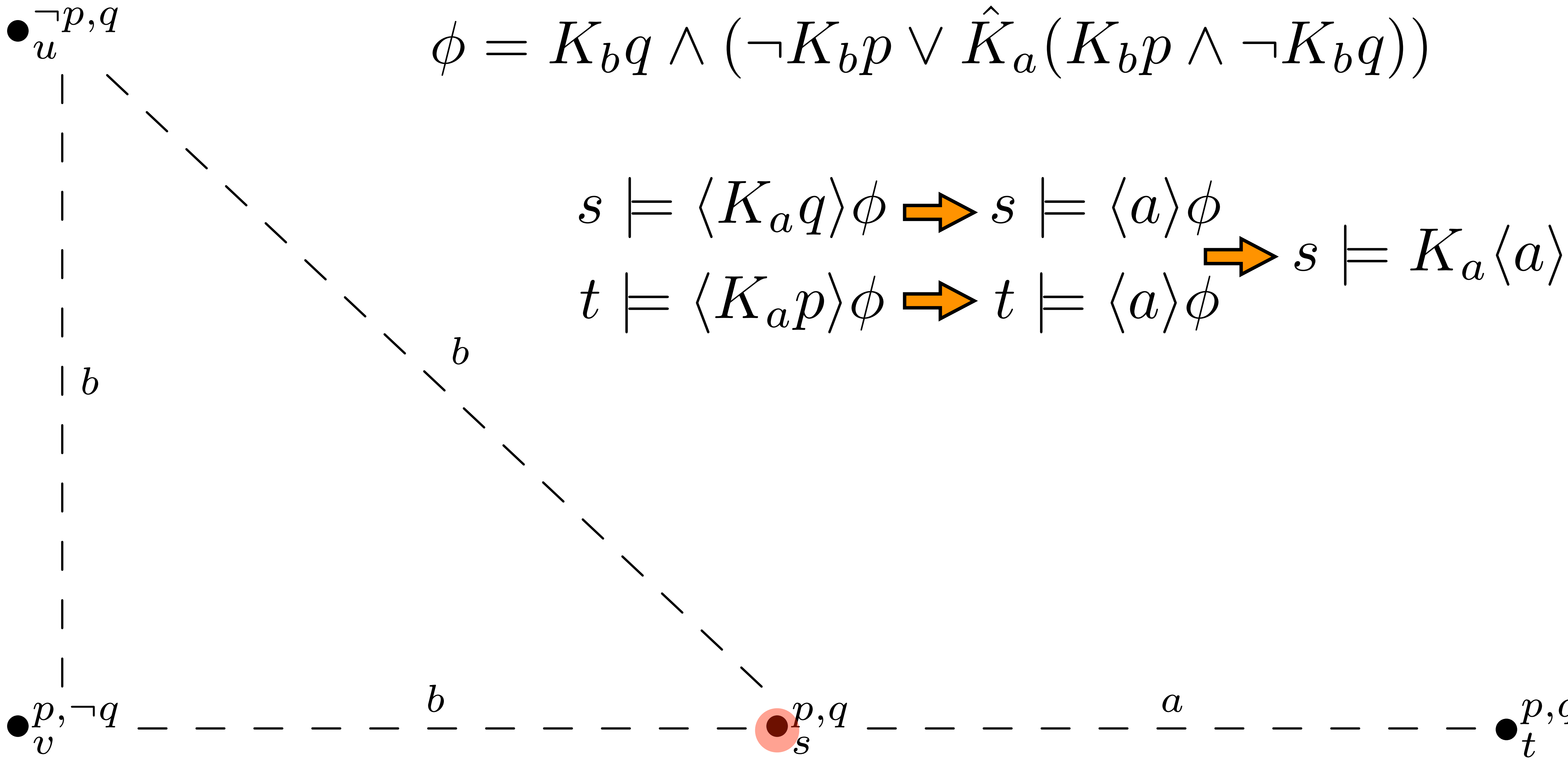

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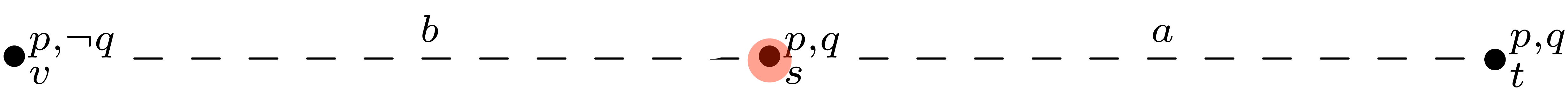


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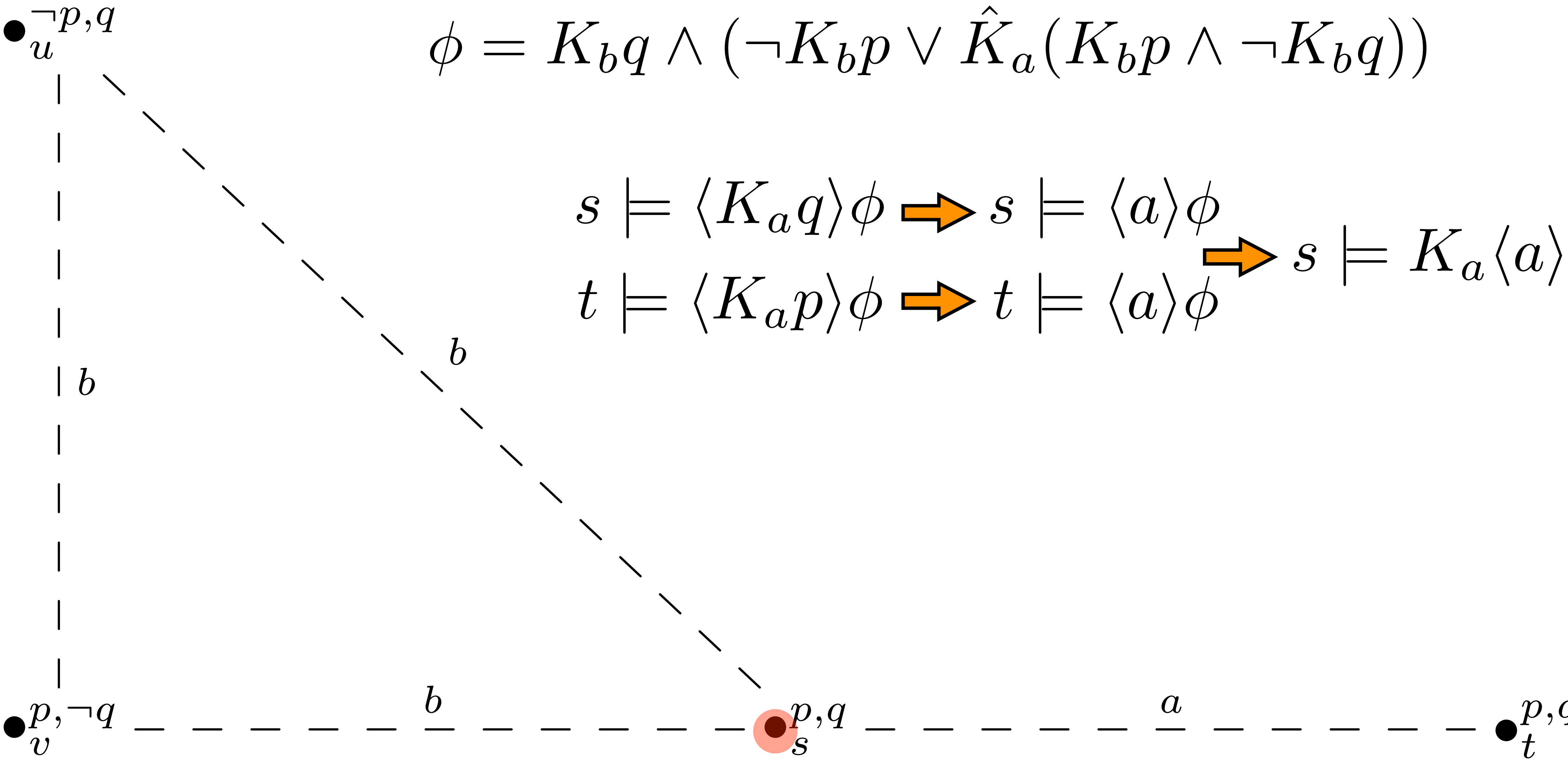

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$$s \not\models \langle K_a p\rangle\phi$$

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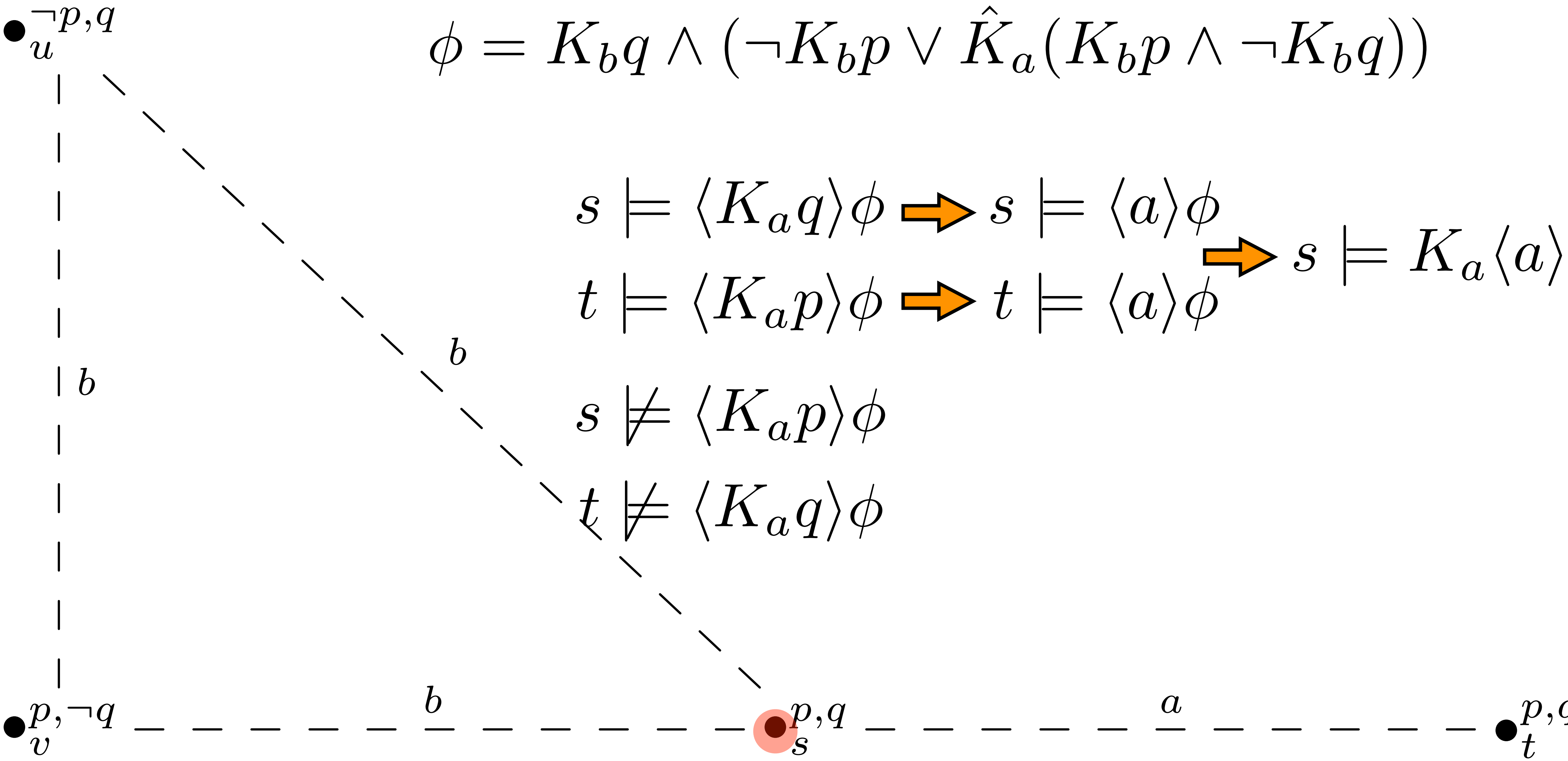

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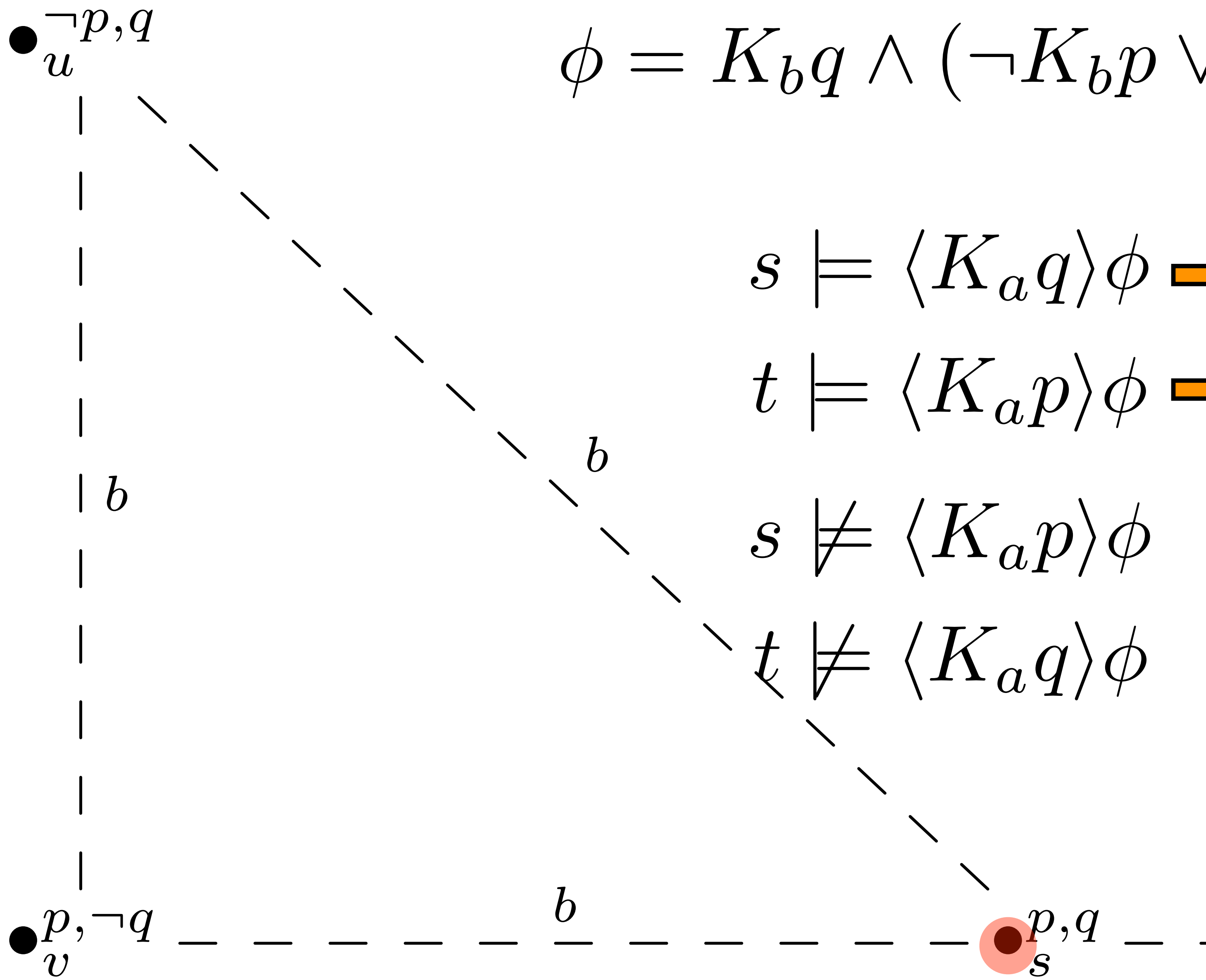


$s \models \langle K_a q \rangle \phi \Rightarrow s \models \langle a \rangle \phi$

$t \models \langle K_a p \rangle \phi \Rightarrow t \models \langle a \rangle \phi$

$\Rightarrow s \models K_a \langle a \rangle \phi$

$s \not\models \langle K_a p \rangle \phi$

$t \not\models \langle K_a q \rangle \phi$

**The same announcement will not achieve the goal in both s and t - *a* does not know *how* to achieve it**

# Expressing knowledge *de dicto*/*de re*

| Ability | Knowledge of ability, *de dicto* | Knowledge of ability, *de re* |
|---|---|---|
| $\exists\psi\ s \models \langle K_a\psi\rangle\phi$ | $\forall s \sim_a t\ \exists\psi\ t \models \langle K_a\psi\rangle\phi$ | $\exists\psi\ \forall s \sim_a t\ t \models \langle K_a\psi\rangle\phi$ |
| $\Updownarrow$ | $\Updownarrow$ | $\Updownarrow$ |
| $s \models \langle a\rangle\phi$ | $s \models K_a\langle a\rangle\phi$ | $??$ |

# Expressing knowledge *de dicto/de re*

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| --- | --- | --- |
| $\exists\psi\ s \models \langle K_a\psi\rangle\phi$ | $\forall s \sim_a t\ \exists\psi\ t \models \langle K_a\psi\rangle\phi$ | $\exists\psi\ \forall s \sim_a t\ t \models \langle K_a\psi\rangle\phi$ |
| $\Updownarrow$ | $\Updownarrow$ | $\Updownarrow$ |
| $s \models \langle a\rangle\phi$ | $s \models K_a\langle a\rangle\phi$ | ?? |

$$s \models \langle a\rangle K_a\phi$$

$$\Updownarrow$$

$$\exists\psi\ s \models \langle K_a\psi\rangle K_a\phi$$

# Expressing knowledge *de dicto*/*de re*

**Ability**

$$\exists \psi \;\; s \models \langle K_a \psi \rangle \phi$$

$$\Updownarrow$$

$$s \models \langle a \rangle \phi$$

**Knowledge of ability, *de dicto***

$$\forall s \sim_a t \;\; \exists \psi \;\; t \models \langle K_a \psi \rangle \phi$$

$$\Updownarrow$$

$$s \models K_a \langle a \rangle \phi$$

**Knowledge of ability, *de re***

$$\exists \psi \;\; \forall s \sim_a t \;\; t \models \langle K_a \psi \rangle \phi$$

$$\Updownarrow$$

$$s \models \langle a \rangle K_a \phi$$

$$\Updownarrow$$

$$\exists \psi \;\; s \models \langle K_a \psi \rangle K_a \phi$$

# Expressing knowledge *de dicto/de re*

| Ability | Knowledge of ability, *de dicto* | Knowledge of ability, *de re* |
| --- | --- | --- |
| $\exists\psi \; s \models \langle K_a\psi\rangle\phi$ | $\forall s \sim_a t \; \exists\psi \; t \models \langle K_a\psi\rangle\phi$ | $\exists\psi \; \forall s \sim_a t \; t \models \langle K_a\psi\rangle\phi$ |
| $\Updownarrow$ | $\Updownarrow$ | $\Updownarrow$ |
| $s \models \langle a\rangle\phi$ | $s \models K_a\langle a\rangle\phi$ | $s \models \langle a\rangle K_a\phi$ |
| | | $\Updownarrow$ |
| | | $\exists\psi \; s \models \langle K_a\psi\rangle K_a\phi$ |

**Depends on**
**(1) the fact that actions are *announcements***
**(2) the S5 properties**

# Example: **Russian Cards** (ctnd.)

Ann and Bill *knows how* to exectute a successful protocol:

$$\langle a \rangle K_a(two \wedge three \wedge \langle b \rangle K_b(one \wedge two \wedge three))$$

# Some logical properties

$$[G \cup H]\phi \to [G][H]\phi$$

$$\langle G\rangle[G]\phi \to [G]\langle G\rangle\phi$$ (Church-Rosser)

$$\langle G\rangle[H]\phi \to [H]\langle G\rangle\phi$$ (..generalised)

# Axiomatisation

$S5_n$ axioms and rules

$PAL$ axioms and rules

$$[G]\phi \rightarrow [\bigwedge_{i \in G} K_i \psi_i]\phi \qquad \text{where } \psi_i \in \mathcal{L}_{el}$$

From $\phi$, infer $[G]\phi$

From $\phi \rightarrow [\theta][\bigwedge_{i \in G} K_i p_i]\psi$, infer $\phi \rightarrow [\theta][G]\psi$
where $p_i \notin \Theta_\phi \cup \Theta_\theta \cup \Theta_\psi$

**Theorem:**
Sound & complete.

# Model Checking

The model checking problem:

$$\text{Given } M, s \text{ and } \phi\text{, does } M, s \models \phi \text{ hold?}$$

**Theorem:**
The model checking problem is PSPACE-complete

(also extends to APAL)

# Decidability

# Motivation and overview

When adding logical dynamics to a decidable ”static” (e.g., epistemic) logic, it is highly unpredictable wether the resulting logic is decidable.

Several open problems concerning the decidability of dynamic epistemic logics.

# Background: Group Announcement Logic

$M^\psi = (S', \sim', V')$ is such that:

- $S' = \{s \in S \mid M_s \models \psi\}$;
- for all $a \in A$, $\sim'_a = \sim_a \cap (S' \times S')$;
- for all $p \in P$, $V'(p) = V(p) \cap S'$.

Interpretation:

$$
\begin{array}{rcl}
M_s \models p & \text{iff} & s \in V(p) \\
M_s \models \neg\phi & \text{iff} & M_s \not\models \phi \\
M_s \models \phi_1 \wedge \phi_2 & \text{iff} & M_s \models \phi_1 \text{ and } M_s \models \phi_2 \\
M_s \models K_a\phi & \text{iff} & \forall t \in S \text{ where } s \sim_a t,\ M_t \models \phi \\
M_s \models [\psi]\phi & \text{iff} & M_s \models \psi \Longrightarrow M_s^\psi \models \phi \\
M_s \models [G]\phi & \text{iff} & \forall \{\psi_a \in \mathcal{L}_{el} : a \in G\},\ M_s \models [\bigwedge_{a \in G} K_a\psi_a]\phi
\end{array}
$$

GAL is finitely axiomatisable.

$\mathcal{L}_{el}$: the purely epistemic language

# Background: Group Announcement Logic

$M^{\psi} = (S', \sim', V')$ is such that:

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- $V'(p) = V(p) \cap S'$.

Interpretation:

$$M_s \models p \quad \text{iff} \quad s \in V(p)$$

Arbitrary Public Announcement Logic (APAL) (Balbiani et al. 2008):

$$M_s \models \Box\phi \quad \text{iff} \quad \forall \psi \in \mathcal{L}_{el},\ M_s \models [\psi]\phi$$

Known to be undecidable (French and van Ditmarsch, 2008)

$$M_s \models [G]\phi \quad \text{iff} \quad \forall \{\psi_a \in \mathcal{L}_{el} : a \in G\},\ M_s \models [\bigwedge_{a \in G} K_a \psi_a]\phi$$

GAL is finitely axiomatisable.

$\mathcal{L}_{el}$: the purely epistemic language

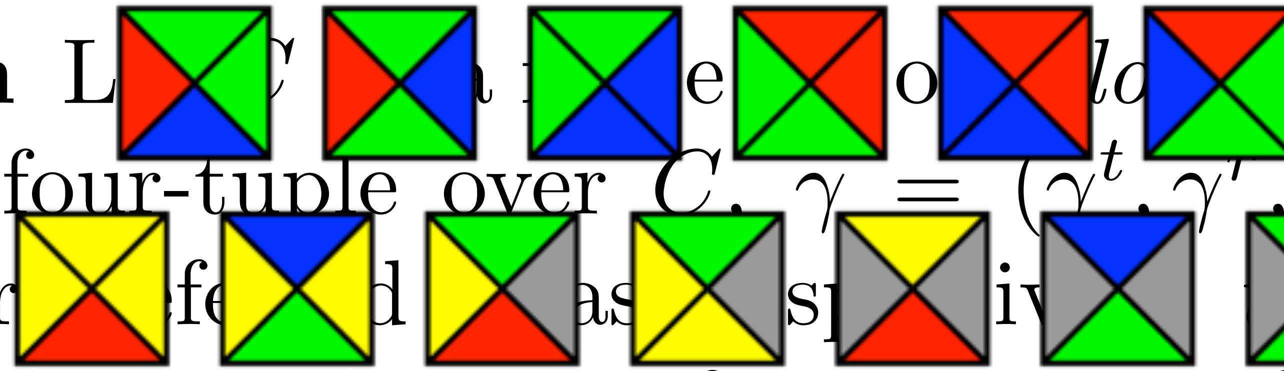
# Tilings and undecidability

**Definition** Let $C$ be a finite set of *colours* and define a $C$-tile $\gamma$ to be a four-tuple over $C$, $\gamma = (\gamma^t, \gamma^r, \gamma^f, \gamma^\ell)$, where the elements are referred to as, respectively, *top*, *right*, *floor* and *left*. The tiling problem is, for any given finite set of $C$-tiles, $\Gamma$, determine if there is a function $\lambda : \mathbb{Z} \times \mathbb{Z} \longrightarrow \Gamma$ such that for all $(i, j) \in \mathbb{Z} \times \mathbb{Z}$:

1. $\lambda(i, j)^t = \lambda(i, j + 1)^f$
2. $\lambda(i, j)^r = \lambda(i + 1, j)^\ell$.

This variant of the tiling problem is known to be undecidable (Harel, 1986).

# Tilings and undecidability



**Definition** Let $C$ be a finite set of *colours* and define a $C$-tile $\gamma$ to be a four-tuple over $C$, $\gamma = (\gamma^t, \gamma^r, \gamma^f, \gamma^\ell)$, where the elements are referred to as, respectively, the *top*, *right*, *floor* and *left*. The tiling problem is, for any given finite set of $C$-tiles, $\Gamma$, determine if there is a function $\lambda : \mathbb{Z} \times \mathbb{Z} \longrightarrow \Gamma$ such that for all $(i, j) \in \mathbb{Z} \times \mathbb{Z}$:

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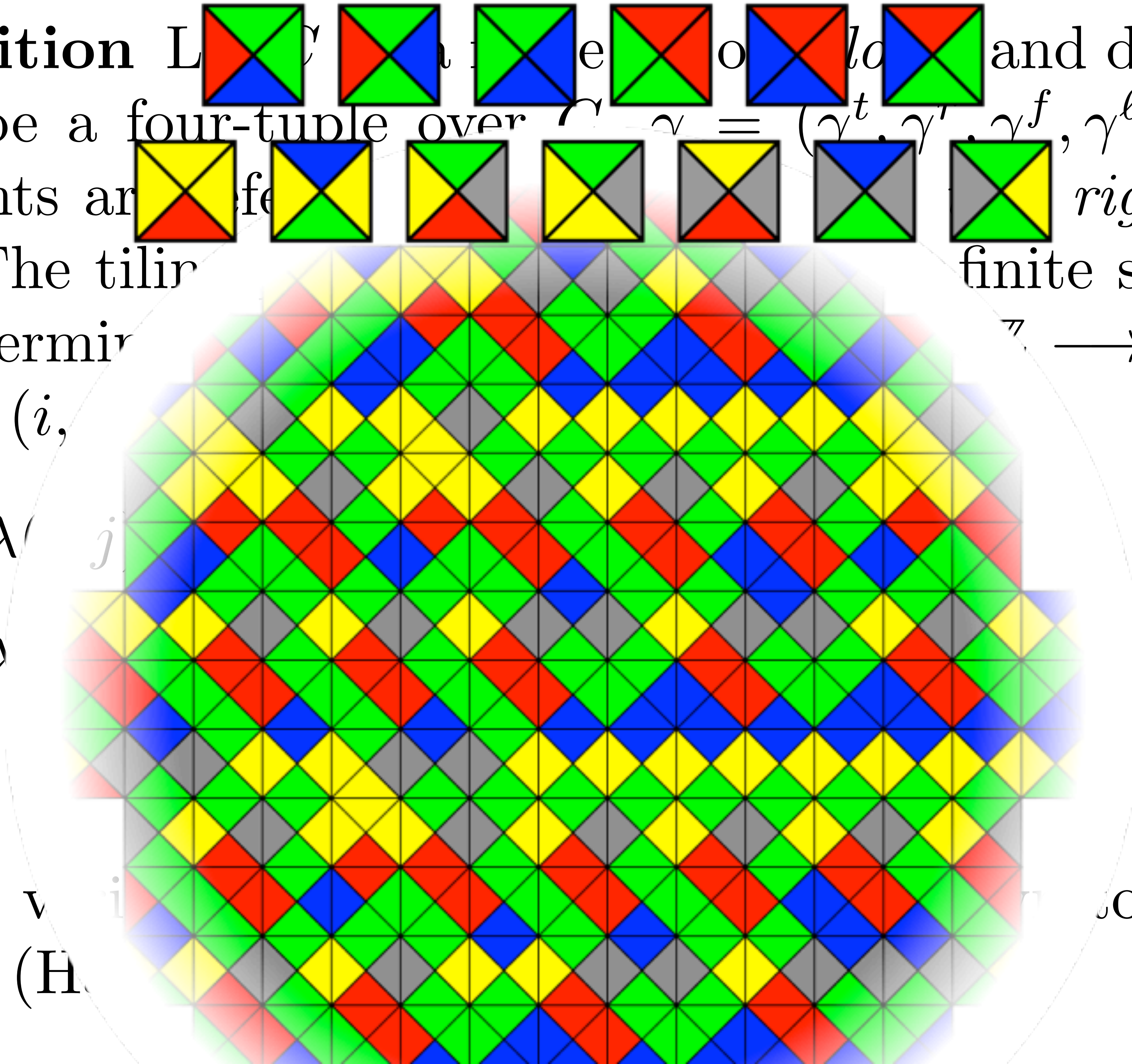
# Tilings and undecidability

**Definition** L... $C$ ... a ... e ... o ... *lo*... and define a $C$-tile $\gamma$ to be a four-tuple over $C$, $\gamma = (\gamma^t, \gamma^r, \gamma^f, \gamma^\ell)$, where the elements ar... efe... *right*, *floor* and *left*. The tilin... finite set of $C$-tiles, $\Gamma$, determin... $\mathbb{Z} \longrightarrow \Gamma$ such that for all $(i,$ ...



1. $\lambda($ ... $j$ ...

2. $\lambda$ ...

This w... i ... o be undecid-able (H...

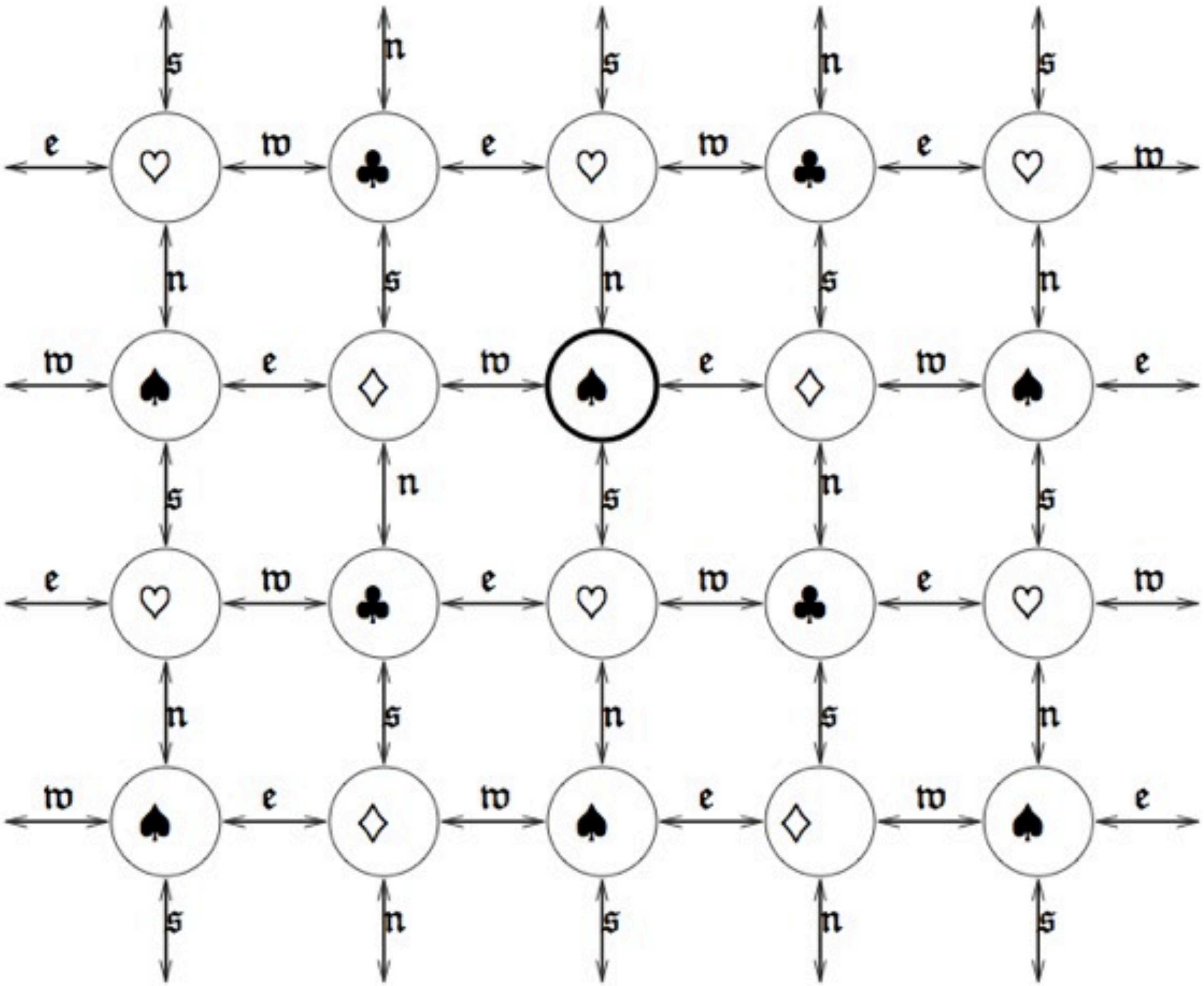
# Undecidability proof: overview

Main steps:

1. enforcing the structure of a satisfying model to have a grid-like structure;
2. defining a formula to represent common knowledge;
3. using propositional atoms to represent tiles, express the formula "it is common knowledge that adjacent tiles on the grid have matching sides".

Similar path as existing undecidability proof for APAL (French and van Ditmarsch, 2008)

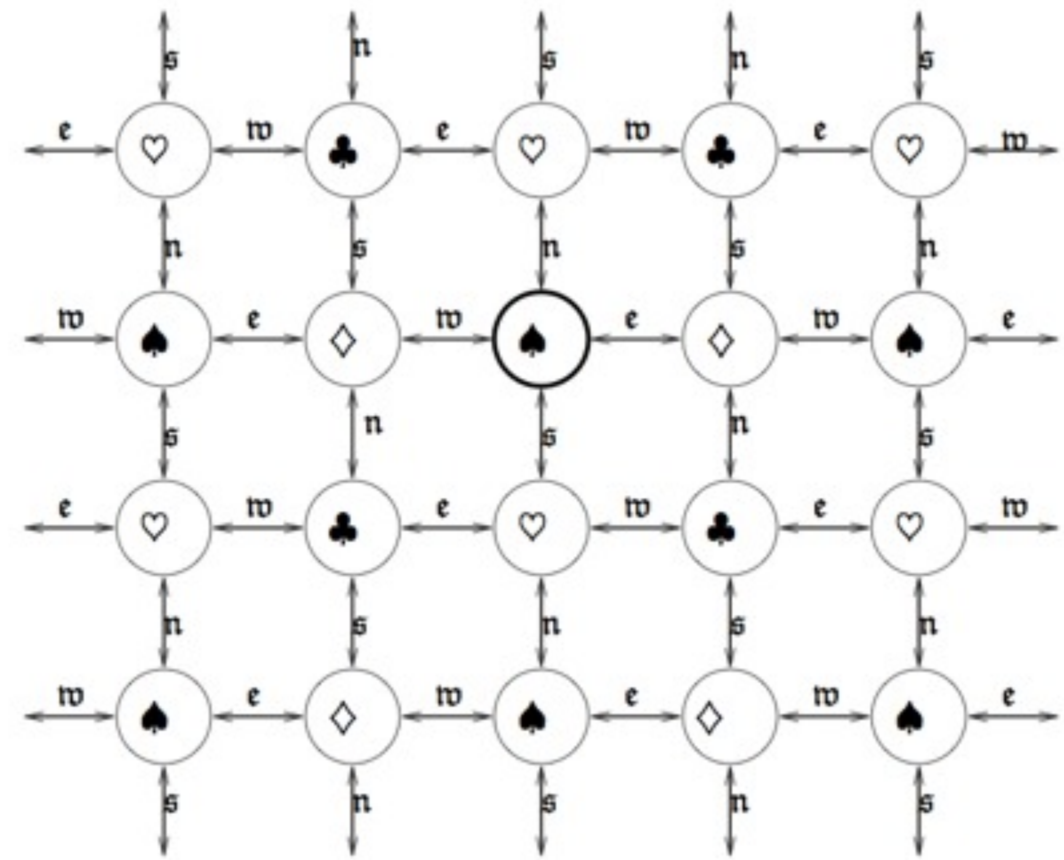
# Grid-like structures



Given: a set of tiles $\Gamma$

- 5 agents: *East* ($\mathfrak{e}$), *West* ($\mathfrak{w}$), *North* ($\mathfrak{n}$), *South* ($\mathfrak{s}$), and one agent that simulates the common knowledge of the other agents ($\mathfrak{t}$).
- Atomic propositions:
  - $\heartsuit$, $\clubsuit$, $\diamondsuit$ and $\spadesuit$
  - $p_\gamma$, for each $\gamma \in \Gamma$

# Tiling grid-like structures



$$up_\Gamma = \bigwedge_{\gamma\in\Gamma}\left(p_\gamma \to \bigvee_{\substack{\delta\,\in\,\Gamma\\ \gamma^t\,=\,\delta^f}} \bigwedge \begin{bmatrix} \heartsuit \to K_{\mathfrak{s}}(\spadesuit \to p_\delta) \\ \clubsuit \to K_{\mathfrak{n}}(\diamondsuit \to p_\delta) \\ \diamondsuit \to K_{\mathfrak{s}}(\clubsuit \to p_\delta) \\ \spadesuit \to K_{\mathfrak{n}}(\heartsuit \to p_\delta) \end{bmatrix}\right)$$

$$down_\Gamma = \bigwedge_{\gamma\in\Gamma}\left(p_\gamma \to \bigvee_{\substack{p_\delta\,\in\,\Gamma\\ \gamma^f\,=\,p_\delta^t}} \bigwedge \begin{bmatrix} \heartsuit \to K_{\mathfrak{n}}(\spadesuit \to p_\delta) \\ \clubsuit \to K_{\mathfrak{s}}(\diamondsuit \to p_\delta) \\ \diamondsuit \to K_{\mathfrak{n}}(\clubsuit \to p_\delta) \\ \spadesuit \to K_{\mathfrak{s}}(\heartsuit \to p_\delta) \end{bmatrix}\right)$$

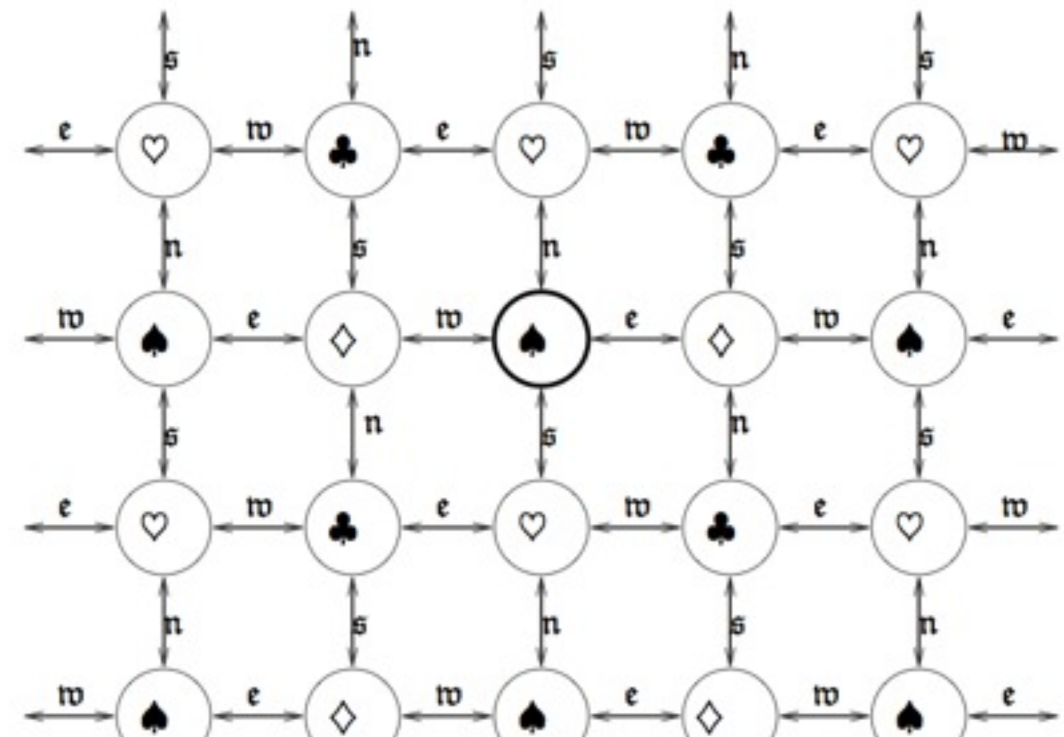
$$left_\Gamma = \bigwedge_{\gamma\in\Gamma}\left(p_\gamma \to \bigvee_{\substack{p_\delta\,\in\,\Gamma\\ \gamma^\ell\,=\,p_\delta^r}} \bigwedge \begin{bmatrix} \heartsuit \to K_{\mathfrak{w}}(\clubsuit \to p_\delta) \\ \clubsuit \to K_{\mathfrak{e}}(\heartsuit \to p_\delta) \\ \diamondsuit \to K_{\mathfrak{w}}(\spadesuit \to p_\delta) \\ \spadesuit \to K_{\mathfrak{e}}(\diamondsuit \to p_\delta) \end{bmatrix}\right)$$

$$right_\Gamma = \bigwedge_{\gamma\in\Gamma}\left(p_\gamma \to \bigvee_{\substack{p_\delta\,\in\,\Gamma\\ \gamma^r\,=\,p_\delta^\ell}} \bigwedge \begin{bmatrix} \heartsuit \to K_{\mathfrak{e}}(\clubsuit \to p_\delta) \\ \clubsuit \to K_{\mathfrak{w}}(\heartsuit \to p_\delta) \\ \diamondsuit \to K_{\mathfrak{e}}(\spadesuit \to p_\delta) \\ \spadesuit \to K_{\mathfrak{w}}(\diamondsuit \to p_\delta) \end{bmatrix}\right)$$

$$unit_\Gamma = \bigvee_{\gamma\in\Gamma} p_\gamma \wedge \bigwedge_{\gamma\in\Gamma}\left(p_\gamma \to \bigwedge_{\delta\in\Gamma\setminus\{\gamma\}} \neg p_\delta\right)$$

$$Tile_\Gamma = K_{\mathfrak{t}}(up_\Gamma \wedge down_\Gamma \wedge left_\Gamma \wedge right_\Gamma \wedge unit_\Gamma)$$

# Tiling grid-like structures



$$up_\Gamma = \bigwedge_{\gamma\in\Gamma}\left(p_\gamma \to \bigvee_{\substack{\delta\,\in\,\Gamma\\ \gamma^t\,=\,\delta^f}} \bigwedge \begin{bmatrix} \heartsuit \to K_{\mathfrak{s}}(\spadesuit\to p_\delta)\\ \clubsuit\to K_{\mathfrak{n}}(\diamondsuit\to p_\delta)\\ \diamondsuit\to K_{\mathfrak{s}}(\clubsuit\to p_\delta)\\ \spadesuit\to K_{\mathfrak{n}}(\heartsuit\to p_\delta)\end{bmatrix}\right)$$

$$down_\Gamma = \bigwedge \left(p_\gamma \to \bigvee \bigwedge \begin{bmatrix} \heartsuit \to K_{\mathfrak{n}}(\spadesuit\to p_\delta)\\ \clubsuit\to K_{\mathfrak{s}}(\diamondsuit\to p_\delta)\\ \diamondsuit\to K \end{bmatrix}\right)$$

**Lemma 1** *A grid-like model satisfies the formula* $Tile_\Gamma$ *if and only if* $\Gamma$ *can tile the integer plane.*

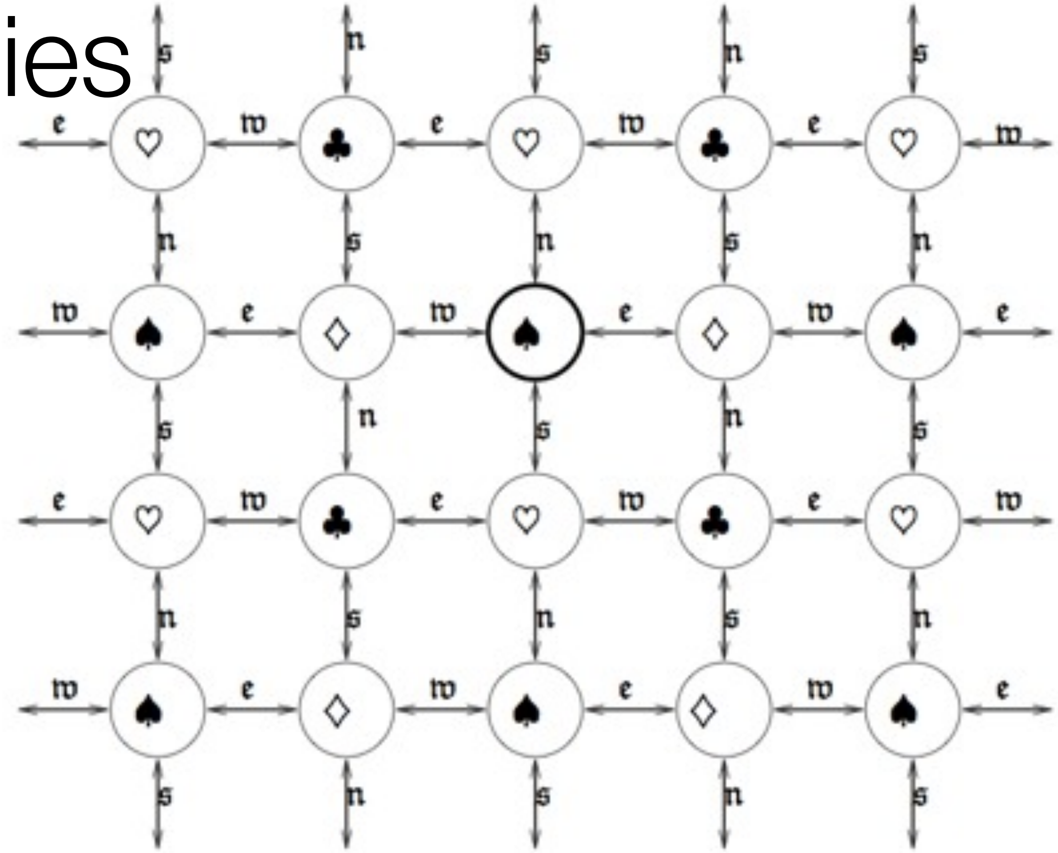
$$\bigwedge_{\gamma\in\Gamma}\left( \bigvee_{\substack{p_\delta\,\in\,\Gamma\\ \gamma^\ell\,=\,p_\delta^r}} \bigwedge \begin{bmatrix} \diamondsuit\to K_{\mathfrak{w}}(\clubsuit\to p_\delta)\\ \spadesuit\to K_{\mathfrak{e}}(\diamondsuit\to p_\delta)\end{bmatrix}\right)$$

$$right_\Gamma = \bigwedge_{\gamma\in\Gamma}\left(p_\gamma \to \bigvee_{\substack{p_\delta\,\in\,\Gamma\\ \gamma^r\,=\,p_\delta^\ell}} \bigwedge \begin{bmatrix} \heartsuit \to K_{\mathfrak{e}}(\clubsuit\to p_\delta)\\ \clubsuit\to K_{\mathfrak{w}}(\heartsuit\to p_\delta)\\ \diamondsuit\to K_{\mathfrak{e}}(\spadesuit\to p_\delta)\\ \spadesuit\to K_{\mathfrak{w}}(\diamondsuit\to p_\delta)\end{bmatrix}\right)$$

$$unit_\Gamma = \bigvee_{\gamma\in\Gamma} p_\gamma \wedge \bigwedge_{\gamma\in\Gamma}\left(p_\gamma \to \bigwedge_{\delta\in\Gamma\setminus\{\gamma\}} \neg p_\delta\right)$$

$$Tile_\Gamma = K_{\mathfrak{t}}(up_\Gamma \wedge down_\Gamma \wedge left_\Gamma \wedge right_\Gamma \wedge unit_\Gamma)$$

# Enforcing the grid: local properties



$$E_{\mathfrak{n}} = \left( \begin{array}{c} L_{\mathfrak{n}}\spadesuit \wedge L_{\mathfrak{n}}\heartsuit \wedge K_{\mathfrak{n}}(\spadesuit \vee \heartsuit) \\ \vee \\ L_{\mathfrak{n}}\clubsuit \wedge L_{\mathfrak{n}}\diamondsuit \wedge K_{\mathfrak{n}}(\clubsuit \vee \diamondsuit) \end{array} \right)$$

$$E_{\mathfrak{s}} = \left( \begin{array}{c} L_{\mathfrak{s}}\spadesuit \wedge L_{\mathfrak{s}}\heartsuit \wedge K_{\mathfrak{s}}(\spadesuit \vee \heartsuit) \\ \vee \\ L_{\mathfrak{s}}\clubsuit \wedge L_{\mathfrak{s}}\diamondsuit \wedge K_{\mathfrak{s}}(\clubsuit \vee \diamondsuit) \end{array} \right)$$

$$E_{\mathfrak{e}} = \left( \begin{array}{c} L_{\mathfrak{e}}\spadesuit \wedge L_{\mathfrak{e}}\diamondsuit \wedge K_{\mathfrak{e}}(\spadesuit \vee \diamondsuit) \\ \vee \\ L_{\mathfrak{e}}\clubsuit \wedge L_{\mathfrak{e}}\heartsuit \wedge K_{\mathfrak{e}}(\clubsuit \vee \heartsuit) \end{array} \right)$$

$$E_{\mathfrak{w}} = \left( \begin{array}{c} L_{\mathfrak{w}}\spadesuit \wedge L_{\mathfrak{w}}\diamondsuit \wedge K_{\mathfrak{w}}(\spadesuit \vee \diamondsuit) \\ \vee \\ L_{\mathfrak{w}}\clubsuit \wedge L_{\mathfrak{w}}\heartsuit \wedge K_{\mathfrak{w}}(\clubsuit \vee \heartsuit) \end{array} \right)$$

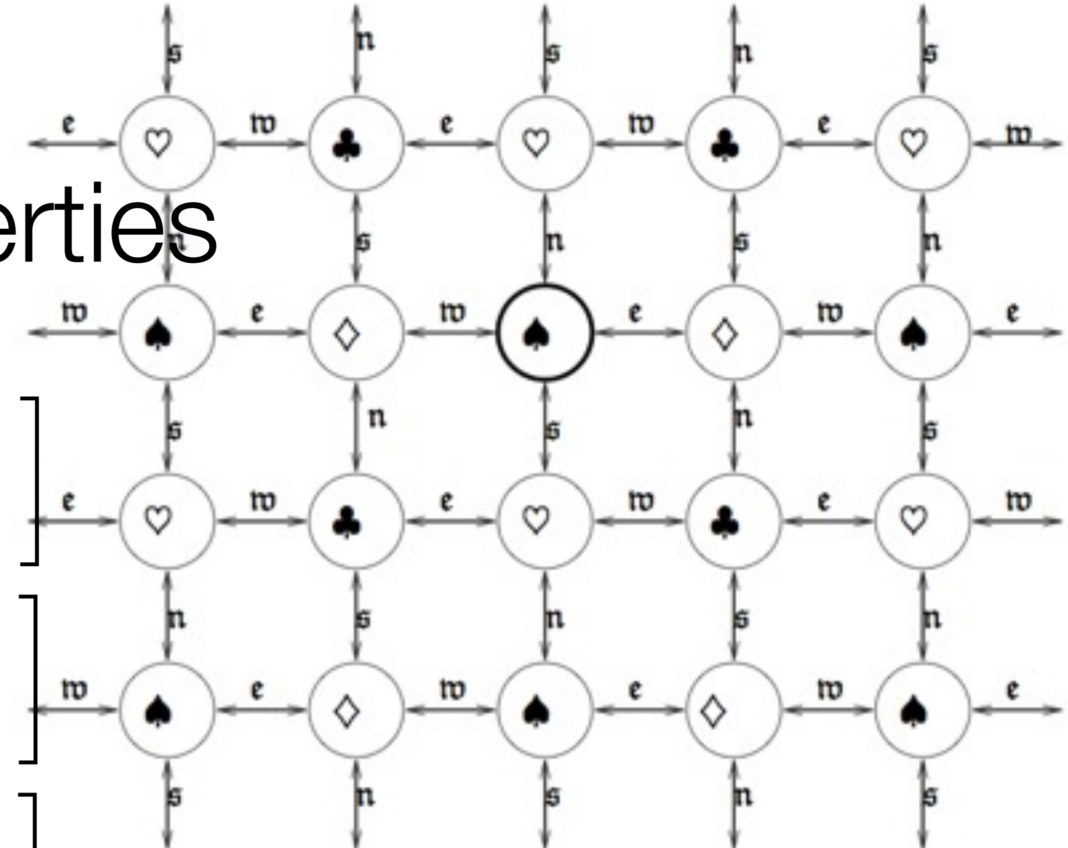
$$E_{\mathfrak{t}} = \left( \begin{array}{c} L_{\mathfrak{t}}\heartsuit \wedge L_{\mathfrak{t}}\clubsuit \wedge L_{\mathfrak{t}}\diamondsuit \wedge L_{\mathfrak{t}}\spadesuit \\ \wedge \\ (\heartsuit \vee \clubsuit \vee \diamondsuit \vee \spadesuit) \end{array} \right)$$

$$L_a\phi = \neg K_a \neg \phi$$

$$Edge = K_{\mathfrak{t}}(E_{\mathfrak{n}} \wedge E_{\mathfrak{s}} \wedge E_{\mathfrak{e}} \wedge E_{\mathfrak{w}} \wedge E_{\mathfrak{t}})$$

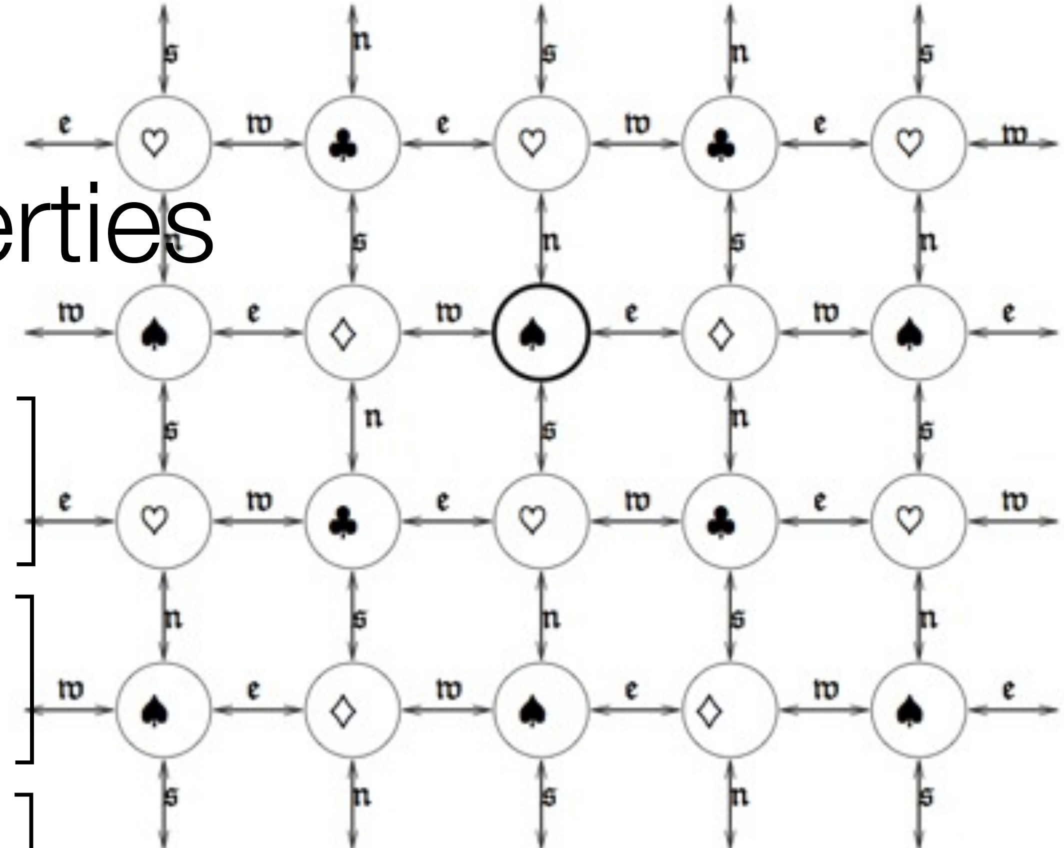
$$Sep = K_{\mathfrak{t}}\Big(\bigvee_{d \in D} \big(d \wedge \bigwedge_{f \in D \setminus d} \neg f\big)\Big) \qquad (D = \{\heartsuit, \clubsuit, \diamondsuit, \spadesuit\})$$

# Enforcing the grid: global properties



$$
\begin{array}{rcl}
G^{\mathfrak{n}}_{\spadesuit} & = & [\{\mathfrak{n},\mathfrak{t}\}] \bigwedge \left[ \begin{array}{l} K_{\mathfrak{e}}(\diamondsuit \to (K_{\mathfrak{s}}(\clubsuit \to L_{\mathfrak{w}}\heartsuit))) \\ K_{\mathfrak{w}}(\diamondsuit \to (K_{\mathfrak{s}}(\clubsuit \to L_{\mathfrak{e}}\heartsuit))) \end{array} \right] \\
G^{\mathfrak{s}}_{\spadesuit} & = & [\{\mathfrak{s},\mathfrak{t}\}] \bigwedge \left[ \begin{array}{l} K_{\mathfrak{e}}(\diamondsuit \to (K_{\mathfrak{n}}(\clubsuit \to L_{\mathfrak{w}}\heartsuit))) \\ K_{\mathfrak{w}}(\diamondsuit \to (K_{\mathfrak{n}}(\clubsuit \to L_{\mathfrak{e}}\heartsuit))) \end{array} \right] \\
G^{\mathfrak{e}}_{\spadesuit} & = & [\{\mathfrak{e},\mathfrak{t}\}] \bigwedge \left[ \begin{array}{l} K_{\mathfrak{n}}(\heartsuit \to (K_{\mathfrak{w}}(\clubsuit \to L_{\mathfrak{s}}\diamondsuit))) \\ K_{\mathfrak{s}}(\heartsuit \to (K_{\mathfrak{w}}(\clubsuit \to L_{\mathfrak{n}}\heartsuit))) \end{array} \right] \\
G^{\mathfrak{w}}_{\spadesuit} & = & [\{\mathfrak{w},\mathfrak{t}\}] \bigwedge \left[ \begin{array}{l} K_{\mathfrak{n}}(\heartsuit \to (K_{\mathfrak{e}}(\clubsuit \to L_{\mathfrak{s}}\diamondsuit))) \\ K_{\mathfrak{s}}(\heartsuit \to (K_{\mathfrak{e}}(\clubsuit \to L_{\mathfrak{n}}\diamondsuit))) \end{array} \right] \\
G_{\spadesuit} & = & \spadesuit \to G^{\mathfrak{n}}_{\spadesuit} \wedge G^{\mathfrak{s}}_{\spadesuit} \wedge G^{\mathfrak{e}}_{\spadesuit} \wedge G^{\mathfrak{w}}_{\spadesuit}
\end{array}
$$

... and similarly for $G_{\heartsuit}$, $G_{\diamond}$ and $G_{\clubsuit}$.



# Enforcing the grid: global properties

$$G^{\mathfrak{n}}_{\spadesuit} = [\{\mathfrak{n},\mathfrak{t}\}] \bigwedge \begin{bmatrix} K_{\mathfrak{e}}(\diamondsuit \to (K_{\mathfrak{s}}(\clubsuit \to L_{\mathfrak{w}}\heartsuit))) \\ K_{\mathfrak{w}}(\diamondsuit \to (K_{\mathfrak{s}}(\clubsuit \to L_{\mathfrak{e}}\heartsuit))) \end{bmatrix}$$

$$G^{\mathfrak{s}}_{\spadesuit} = [\{\mathfrak{s},\mathfrak{t}\}] \bigwedge \begin{bmatrix} K_{\mathfrak{e}}(\diamondsuit \to (K_{\mathfrak{n}}(\clubsuit \to L_{\mathfrak{w}}\heartsuit))) \\ K_{\mathfrak{w}}(\diamondsuit \to (K_{\mathfrak{n}}(\clubsuit \to L_{\mathfrak{e}}\heartsuit))) \end{bmatrix}$$

$$G^{\mathfrak{e}}_{\spadesuit} = [\{\mathfrak{e},\mathfrak{t}\}] \bigwedge \begin{bmatrix} K_{\mathfrak{n}}(\heartsuit \to (K_{\mathfrak{w}}(\clubsuit \to L_{\mathfrak{s}}\diamondsuit))) \\ K_{\mathfrak{s}}(\heartsuit \to (K_{\mathfrak{w}}(\clubsuit \to L_{\mathfrak{n}}\heartsuit))) \end{bmatrix}$$

$$G^{\mathfrak{w}}_{\spadesuit} = [\{\mathfrak{w},\mathfrak{t}\}] \bigwedge \begin{bmatrix} K_{\mathfrak{n}}(\heartsuit \to (K_{\mathfrak{e}}(\clubsuit \to L_{\mathfrak{s}}\diamondsuit))) \\ K_{\mathfrak{s}}(\heartsuit \to (K_{\mathfrak{e}}(\clubsuit \to L_{\mathfrak{n}}\diamondsuit))) \end{bmatrix}$$

$$G_{\spadesuit} = \spadesuit \to G^{\mathfrak{n}}_{\spadesuit} \wedge G^{\mathfrak{s}}_{\spadesuit} \wedge G^{\mathfrak{e}}_{\spadesuit} \wedge G^{\mathfrak{w}}_{\spadesuit}$$

... and similarly for $G_{\heartsuit}$, $G_{\diamond}$ and $G_{\clubsuit}$.

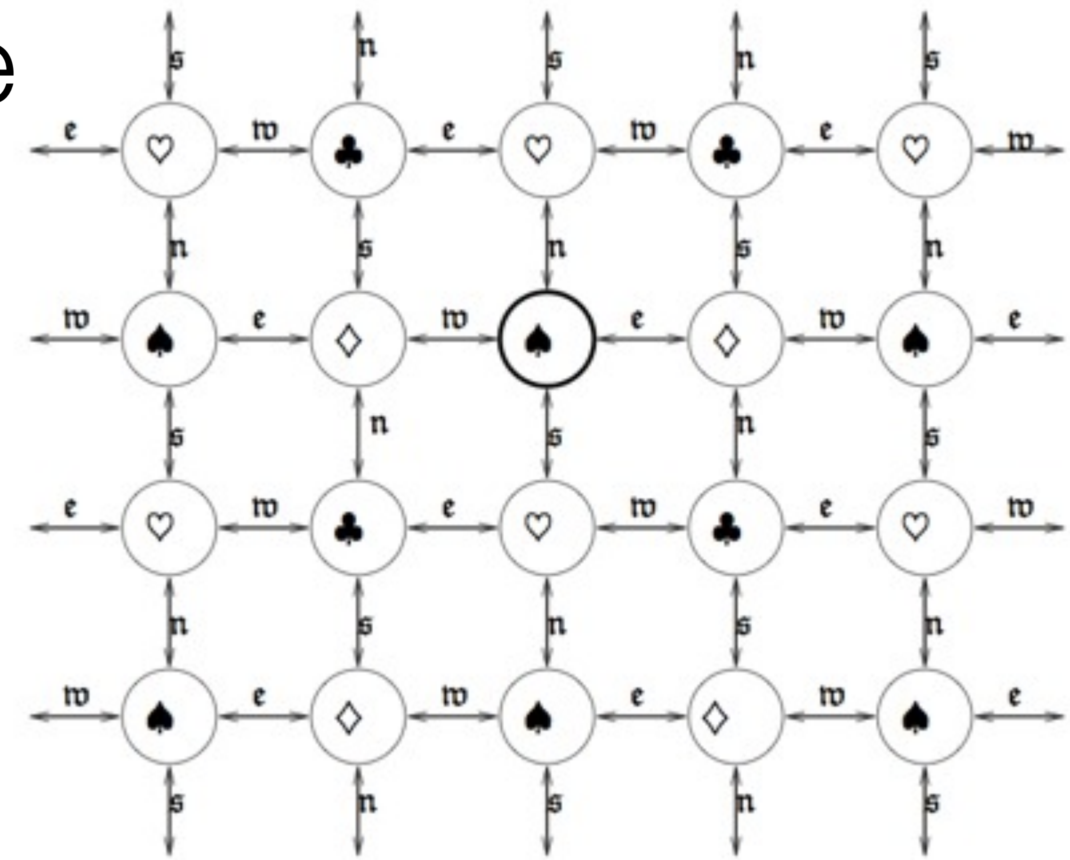
$$G_{\heartsuit} = G(\spadesuit, \diamondsuit, \clubsuit, \heartsuit)$$

$$G_{\clubsuit} = G(\diamondsuit, \spadesuit, \heartsuit, \clubsuit)$$

$$G_{\diamondsuit} = G(\clubsuit, \heartsuit, \spadesuit, \diamondsuit)$$

$$\color{red}{Grid = K_{\mathfrak{t}}(G_{\heartsuit} \wedge G_{\clubsuit} \wedge G_{\diamondsuit} \wedge G_{\spadesuit})}$$

# Simulating common knowledge



$$CK_{\spadesuit} = \spadesuit \to [\{\mathfrak{t}\}](L_{\mathfrak{n}}\heartsuit \wedge L_{\mathfrak{s}}\heartsuit \wedge L_{\mathfrak{e}}\diamondsuit \wedge L_{\mathfrak{w}}\diamondsuit)$$

$$CK_{\heartsuit} = \heartsuit \to [\{\mathfrak{t}\}](L_{\mathfrak{n}}\spadesuit \wedge L_{\mathfrak{s}}\spadesuit \wedge L_{\mathfrak{e}}\clubsuit \wedge L_{\mathfrak{w}}\clubsuit)$$

$$CK_{\clubsuit} = \clubsuit \to [\{\mathfrak{t}\}](L_{\mathfrak{n}}\diamondsuit \wedge L_{\mathfrak{s}}\diamondsuit \wedge L_{\mathfrak{e}}\heartsuit \wedge L_{\mathfrak{w}}\heartsuit)$$

$$CK_{\diamondsuit} = \diamondsuit \to [\{\mathfrak{t}\}](L_{\mathfrak{n}}\clubsuit \wedge L_{\mathfrak{s}}\clubsuit \wedge L_{\mathfrak{e}}\spadesuit \wedge L_{\mathfrak{w}}\spadesuit)$$

$$CK = K_{\mathfrak{t}}(CK_{\spadesuit} \wedge CK_{\heartsuit} \wedge CK_{\clubsuit} \wedge CK_{\diamondsuit})$$

# Putting it all together

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$$T(\Gamma) = Grid \wedge Edge \wedge CK \wedge Tile_{\Gamma} \wedge Sep$$

**Lemma 1** *$T(\Gamma)$ is satisfiable if and only if $\Gamma$ can tile the plane.*

# Putting it all together

$$T(\Gamma) = Grid \wedge Edge \wedge CK \wedge Tile_\Gamma \wedge Sep$$

**Lemma 1** *$T(\Gamma)$ is satisfiable if and only if $\Gamma$ can tile the plane.*

**Theorem 1** *The satisfiability problem for GAL is co-RE complete.*

# Decidability: conclusions and future work

- Satisfiability of GAL is undecidable
- Note that we didn’t use the PAL operators => stronger result
- Proof structure might be interesting for other logics
- What are interesting alternative approaches to reasoning about what agents can achieve by sharing knowledge?
  - Arbitrary Public Announcement Logic (Balbiani et al, 2008). Undecidable.
  - Coalition Announcement Logic (Ågotnes and van Ditmarsch, 2008): undecidable
  - Quantification over refinements (van Ditmarsch et al., 2010) or event models (Hales 2013): both decidable
  - Restrictions of announcements to positive formulae

# General directions

- More general actions/events
- Coalition Announcement Logic
  - a coalition logic
  - there are announcements by G such that for all announcements by the other agents, ...

# Open problems

- Expressive power:
  - It is known that GAL is not as expressive as APAL
  - Unknown: can APAL express everything GAL can express (in the multi-agent case)?
- GAL: decidable fragments?
- CAL:
  - expressive power
  - axiomatisation

# For more details:

- T. Ågotnes and H. van Ditmarsch, *Coalitions and Announcements*, Proc. AAMAS 2008
- T. Ågotnes, P. Balbiani, H. van Ditmarsch and P. Seban, *Group Announcement Logic*, Journal of Applied Logic **8**(1), 2010
- T. Ågotnes, T. French and H. van Ditmarsch, *The Undecidability of Group Announcements,* Proc. AAMAS 2014
- T. Ågotnes, T. French and H. van DItmarsch, *The Undecidability of Quantified Announcements*, Studia Logica, 2016